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TeV-scale Z' bosons from D-branes

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Abstract: Generic D-brane string models of particle physics predict the existence of extra U(1) gauge symmetries beyond hypercharge. These symmetries are not of the E_6 class but rather include the gauging of Baryon and Lepton numbers as well as certain Peccei-Quinn-like symmetries. Some of the U(1)'s have triangle anomalies, but they are cancelled by a Green-Schwarz mechanism. The corresponding gauge bosons typically acquire a mass of order the string scale M_S by combining with two-index antisymmetric fields coming from the closed string sector of the theory. We argue that in string models with a low string scale $M_S \propto 1$–$10\,\mathrm{TeV}$, the presence of these generic U(1)'s may be amenable to experimental test. Present constraints from electroweak precision data already set important bounds on the mass of these extra gauge bosons. In particular, for large classes of models, ρ-parameter constraints imply $M_S \geq 1.5\,\mathrm{TeV}$. In the present scheme some fraction of the experimentally measured Z^0 mass would be due not to the Higgs mechanism, but rather to the mixing with these closed string fields. We give explicit formulae for recently constructed classes of intersecting D6- and D5-brane models yielding the Standard Model (SM) fermion spectrum.

Keywords: D-branes, Compactification and String Models, Extra Large Dimensions, Beyond Standard Model.

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Contents

1. Introduction

One of the aims of string phenomenology is to extract generic features of realistic string models which can be considered as "predictions" that could eventually be tested experimentally. Finding one generic property of string models is particularly relevant, given the large degeneracy of string vacua.

At present there are two general classes of chiral D-brane models with realistic properties, corresponding to D-branes at singularities [1, 2, 3] and D-branes intersecting at non-trivial angles [4]–[14]. Both classes of models share very encouraging phenomenological prospects: in both cases it is possible to construct three-family models resembling very much the structure of the Standard Model (SM). The gauge symmetry always arises from a product of U(N) groups and the matter fields transform generically in bi-fundamental representations of those groups, giving rise to a rich structure that can be encoded in terms of quiver diagrams [12, 15]. Another interesting property of explicit D-brane models is that the proton appears to be generically stable. There are also other characteristics that are more model dependent: the models may be non-supersymmetric [1, 2] and [4]–[14], supersymmetric [2, 3, 9] or "quasi"-supersymmetric [12, 13], the string scale may be as high as the Planck scale and as low as 1 TeV and gauge and Yukawa couplings may be found to take realistic values.

It is then desirable to concentrate on one generic feature of this class of models. In this paper we will study in detail the general implications of having extra U(1) symmetries beyond the Standard Model hypercharge, which is generic in D-brane models [16]. The reason for the appearance of the extra U(1)'s is the following. The basic structure of D-branes implies the existence of a U(1) for each D-brane. Having N overlapping D-branes implies that open string states ending on different D-branes become massless when the separation vanishes and the $\mathrm{U}(1)^N$ symmetry is enhanced to U(N). Therefore, whenever we look for a chiral model including the symmetries of the Standard Model of particle physics, we cannot obtain just the symmetries $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ but instead we will have at least $\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(1)$. This implies that there will necessarily be additional U(1) symmetries beyond the Standard Model and also that these extra U(1)'s are precisely defined to be the ones that complete the SU(N) groups to U(N). If the string scale is sufficiently low, the associated gauge bosons may be relatively light and precision tests of the Standard Model can already set a bound on their masses which in turn may imply a bound on the string scale. In this context notice that signatures of additional Z' bosons can be detected easier than almost any other possible signature beyond the Standard Model [17]–[21].

Most of the extra U(1)'s in D-brane models are usually massive and the mechanism through which they acquire a mass is well understood [22, 23] (for related discussions see [24]–[29]). This is due to the existence of interaction terms of the form $B \wedge F$, where B stands for the two-index antisymmetric tensors present in the closed string sector of the theory. These terms are crucial for the existence of the $D = 4$ Green-Schwarz mechanism for the cancellation of U(1) anomalies. Upon dualization, these terms convert into Stuckelberg mass terms for the corresponding gauge field. Interestingly enough, these couplings have been found [8] not only for "anomalous" U(1)'s but also for non-anomalous ones. The general situation is that only U(1) of hypercharge survives as a massless gauge boson and all the other extra Abelian gauge bosons acquire a mass of the order of the string scale. If this scale is close to 1 TeV then it may be easy to find signatures of those fields. In particular, in the present scheme a fraction of the mass of the Z^0 boson would be due not to the standard Higgs mechanism, but to the mixing with closed string antisymmetric tensor fields B.

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The study of explicit models shows that the additional U(1)'s may correspond to physically relevant symmetries: baryon number, lepton number and a PQ-like symmetry. The interesting aspect is that these symmetries may still survive at low energies as exact global symmetries. The reason for this is the way those fields acquire a mass, which is not the standard Higgs mechanism, since it is not necessarily accompanied by a non-vanishing vev for any scalar field. Thus there is no longer a local invariance (there is no massless U(1) boson) but the global symmetry remains. The survival of the baryon number as an effective global symmetry naturally guarantees the proton stability, which is a serious problem in models with a low string scale.

In this context, a class of explicit type-IIA orientifold models was recently constructed yielding just the fermions of the SM at the intersections of D6-branes wrapping a 6-torus [12, 13]. These brane configurations provide a natural explanation for properties of the SM like family replication, due to the fact that branes wrapped on a compact space typically intersect more than once. More recently, these constructions have been extended to the case of intersecting D5-branes [14]. These brane intersection models provide us for the first time with explicit realistic models in which the structure of U(1) gauge bosons can be analyzed in detail. In these models one can write explicit results for the couplings of the RR-scalars to the U(1) gauge bosons which eventually give Stuckelberg masses to the extra U(1)'s. This is the reason why we will concentrate in the numerical analysis on the specific case of D6- and D5-brane intersection models.

The paper is organised as follows. In the next chapter we start with a general discussion of the mechanism by which two-index antisymmetric fields can provide explicit (Stuckelberg) masses to Abelian fields, without the presence of any Higgs mechanism. Then we review the D-brane intersection scenario in which quarks and leptons appear at intersections of D-branes. We show how in this scenario there are in general three extra U(1)'s, of which two have triangle anomalies cancelled by a Green-Schwarz mechanism. We discuss in turn the case of D6- and D5-brane models and provide explicit results for the couplings of the different antisymmetric B-fields to the relevant Abelian fields.

In section 3 we outline the general approach we will follow in the study of the mass matrix of Abelian gauge bosons before the electroweak symmetry breaking. This is done for some families of D6-brane models as well as D5-brane models. We then describe the method to be used for including the effects of the electroweak symmetry breaking on the U(1) masses.

In section 4 we detail our analysis of the masses of U(1) fields before electroweak symmetry breaking following the strategy of section 3. We provide explicit results for the masses of U(1) fields as well as the constraints induced on the value of the string scale M_S for D6 and D5-brane models. In section 5 the effects of the electroweak symmetry breaking are analysed in detail. The neutral gauge bosons acquire now a mass from a combination of two sources: mixing with antisymmetric B-fields and the standard Higgs mechanism induced by the vevs of electroweak doublets. We show how the presence of the stringy source of mass for the Abelian gauge bosons has an impact on the ρ parameter. Present ρ-parameter constraints imply sizable bounds on the masses of the extra Z' bosons (and hence on the string scale M_S). We analyze in detail some D6-brane and D5-brane classes of models and give corresponding bounds. In section 6 some general comments and conclusions are presented. The Appendix contains some additional formulae referred to in the main text.

2. Extra U(1)'s in D-brane standard model-like models

In this section we address the structure of gauged Abelian symmetries beyond hypercharge in D-brane settings yielding the SM at low energies. As mentioned, explicit string D-brane models yielding three generations of quarks and leptons have been constructed in the last few years. In order to obtain chirality it has been considered the location of the SM stacks of branes to be at some (e.g. Z_N orbifold) singularity or/and settings involving intersecting branes. We will concentrate here on models based on intersecting D6- or D5-branes in which it has been recently shown that models with the massless fermion spectrum of the SM are easy to obtain [8, 14], although we think that much of our results are generalisable to other classes of D-brane models.

Before proceeding to the description of the intersecting D-brane models let us review in more detail how the U(1) gauge bosons acquire a mass from the presence of $B\wedge F$ couplings, which is a generic mechanism in string theory models. We would like to emphasize that there is an important difference from the standard Higgs mechanism in the sense that there is not necessarily a remnant massive scalar field to play the role of the corresponding Higgs boson.

2.1 Green-Schwarz terms and massive U(1)'s

To understand the basis of the mechanism giving masses to the U(1)'s let us consider the following lagrangian coupling an Abelian gauge field A_μ to an antisymmetric tensor $B_{\mu\nu}$:

$$\mathcal{L} = -\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho} - \frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} + \frac{c}{4}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}F_{\rho\sigma}\,, \tag{2.1}$$

where

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}\,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.2}$$

and g, c are arbitrary constants. This corresponds to the kinetic term for the fields $B_{\mu\nu}$ and A_μ together with the $B\wedge F$ term. We will now proceed to dualize this lagrangian in two equivalent ways. First we can re-write it in terms of the (arbitrary) field $H_{\mu\nu\rho}$ imposing the constraint $H = dB$ by the standard introduction of a Lagrange multiplier field η in the following way:

$$\mathcal{L}_0 = -\frac{1}{12}H^{\mu\nu\rho}H_{\mu\nu\rho} - \frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} - \frac{c}{6}\epsilon^{\mu\nu\rho\sigma}H_{\mu\nu\rho}A_\sigma - \frac{c}{6}\eta\epsilon^{\mu\nu\rho\sigma}\partial_\mu H_{\nu\rho\sigma}\,. \tag{2.3}$$

Notice that integrating out η implies $d^*H = 0$ which in turn implies that (locally) $H = dB$ and then we recover (2.1). Alternatively, integrating by parts the last term in (2.3) we are left with a quadratic action for H which we can solve immediately to find

$$H^{\mu\nu\rho} = -c\epsilon^{\mu\nu\rho\sigma}\left(A_\sigma + \partial_\sigma\eta\right). \tag{2.4}$$

Inserting this back into (2.3) we find:

$$\mathcal{L}_A = -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{c^2}{2} \left(A_\sigma + \partial_\sigma \eta\right)^2 \tag{2.5}$$

which is just a mass term for the gauge field A_μ after "eating" the scalar η to acquire a mass $m^2 = g^2 c^2$. Notice that this is similar to the Stückelberg mechanism where we do not need a scalar field with a vacuum expectation value to give a mass to the gauge boson, nor do we have a massive Higgs-like field at the end.

Furthermore, we can understand this mechanism in a dual way in which it is not the gauge field that "eats" a scalar, but the antisymmetric tensor that "eats" the gauge field to gain a mass. This can be seen as follows. Start now with the first order lagrangian:

$$\mathcal{L}'_0 = -\frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \frac{c}{4} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma} + \frac{c}{4} Z_\mu \epsilon^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma} \,, \tag{2.6}$$

where $F^{\mu\nu}$ is now an arbitrary tensor (not determined by (2.2)) and Z_μ is a Lagrange multiplier enabling the condition $F = dA$. Similar to the previous case, integrating over Z_μ gives back the original lagrangian, (2.1) but integrating by parts the last term and solving the quadratic equation for $F_{\mu\nu}$ (now an arbitrary field) gives the dual lagrangian:

$$\mathcal{L}_B = -\frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{g^2 c^2}{4} \left(B_{\mu\nu} + \partial_\mu Z_\nu\right)^2 . \tag{2.7}$$

We can see that this is the lagrangian for a massive two-index antisymmetric tensor, which gains a mass after "eating" the vector Z_μ with the same mass as above $m^2 = g^2 c^2$. This is completely equivalent to the massive vector, notice that in four dimensions a massive vector and a *massive* two-index tensor have the same number of degrees of freedom (3). For a general discussion of massive antisymmetric tensor fields see [30].

We would like to emphasize that this mechanism requires the presence of the Green-Schwarz term $B \wedge F$ but not necessarily the anomaly cancellation term ($\eta F \wedge F$). Therefore as long as a U(1) field has a Green-Schwarz coupling $B \wedge F$, it does not have to be anomalous in order to acquire a mass. In most previous models found in string theory it was generally the case that it was only the anomalous U(1)'s that had a Green-Schwarz term and therefore those were the ones becoming massive. However, in recent models [8, 12, 13] there are non-anomalous U(1)'s that can also become massive. Those U(1) bosons acquire a "topological mass" induced by the Green-Schwarz term with no associated massive scalar field in the spectrum. Note that this is totally different from what happens when a U(1) becomes massive in the standard Higgs mechanism. Indeed in the latter case in addition to the massive gauge boson there is always an explicit scalar field, the Higgs field. Such a field is not present in the mechanism we discussed.

We finally note that as emphasized in [8], the gauge group is broken to a global symmetry and therefore symmetries like baryon and lepton number remain as perturbative global symmetries, providing a simple explanation for proton stability in models with a low string scale.

Label	Multiplicity	Gauge Group	Name
stack a	$N_a = 3$	$\mathrm{SU}(3) \times \mathrm{U}(1)_a$	Baryonic brane
stack b	$N_b = 2$	$\mathrm{SU}(2) \times \mathrm{U}(1)_b$	Left brane
stack c	$N_c = 1$	$\mathrm{U}(1)_c$	Right brane
stack d	$N_d = 1$	$\mathrm{U}(1)_d$	Leptonic brane

Table 1: Brane content yielding the SM spectrum.

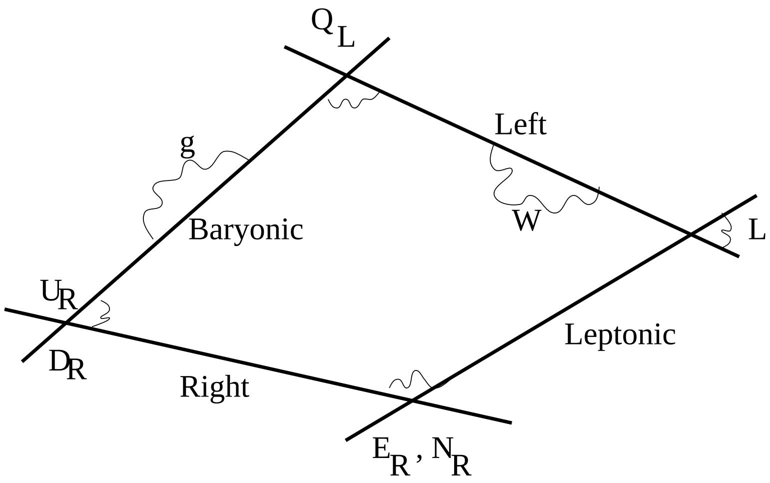

Figure 1: Schematic view of an intersecting D-brane standard model construction. There are four stacks of branes: *Baryonic, Left, Right* and *Leptonic*, giving rise to a gauge group $\mathrm{U}(3)_{Baryonic} \times \mathrm{U}(2)_{Left} \times \mathrm{U}(1)_{Right} \times \mathrm{U}(1)_{Leptonic}$. Open strings starting and ending on the same stack of branes give rise to the SM gauge bosons. Quarks and leptons appear at the intersections of two different stacks of branes.

2.2 U(1) structure in intersecting brane SM-like models

The general structure of intersecting D-brane models is summarized in table 1 and in figure 1.

We have four stacks of branes : the *baryonic* stack contains three parallel branes giving rise to the QCD interactions, and the *left* stack contains two parallel branes yielding the electroweak $\mathrm{SU}(2)_L$ SM interactions. In addition there is the *right* and *lepton* stacks containing each a single brane. These four stacks of branes intersect in the compact six dimensions (plus Minkowski) and at the intersections chiral fermions with the quantum numbers of the SM appear. Thus, for example, the right-handed U-quarks occur at three different intersections of the *baryonic* stack with the *right* stack (see figure 1).

Each stack of branes comes along with a unitary gauge group so that the initial gauge group is $\mathrm{SU}(3)_{QCD} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_a \times \mathrm{U}(1)_b \times \mathrm{U}(1)_c \times \mathrm{U}(1)_d$. A linear combination of these four U(1)'s may be identified with the standard hypercharge and at some level the rest of the U(1)'s should become massive. In the class of D6-brane models of ref. [8] and D5-brane models of ref. [14] the charges of quarks and leptons with respect to these U(1)'s are shown

Intersection	Matter fields		q_a	q_b	q_c	q_d	q_Y
(ab)	Q_L	$(3,2)$	1	-1	0	0	1/6
(ab*)	q_L	$2(3,2)$	1	1	0	0	1/6
(ac)	U_R	$3(\bar{3},1)$	-1	0	1	0	$-2/3$
(ac*)	D_R	$3(\bar{3},1)$	-1	0	-1	0	1/3
(bd*)	L	$3(1,2)$	0	-1	0	-1	$-1/2$
(cd)	E_R	$3(1,1)$	0	0	-1	1	1
(cd*)	N_R	$3(1,1)$	0	0	1	1	0

Table 2: Standard model spectrum and U(1) charges. The hypercharge generator is defined as $q_Y = \frac{1}{6}q_a - \frac{1}{2}q_c + \frac{1}{2}q_d$.

in table 2. In this table the asterisk denotes the "orientifold mirror" of each given brane, which must always be present in this type of orientifold constructions (see [8] for details). Note that $\mathrm{U}(1)_a$ and $\mathrm{U}(1)_d$ can be identified with baryon number and (minus) lepton number, respectively. Additionally $\mathrm{U}(1)_c$ can be identified with the third component of right-handed weak isospin. Finally, $\mathrm{U}(1)_b$ is an axial symmetry with QCD anomalies, very much like a PQ-symmetry. It is easy to check from the above fermion spectrum that $\mathrm{U}(1)_b$ and $3\,\mathrm{U}(1)_a - \mathrm{U}(1)_d$ linear combination have triangle anomalies whereas $\mathrm{U}(1)_a + 3\,\mathrm{U}(1)_d$ and $\mathrm{U}(1)_c$ are both anomaly-free. In fact the standard hypercharge may be written as a linear combination of these two symmetries:

$$q_Y = \frac{1}{6}q_a - \frac{1}{2}q_c + \frac{1}{2}q_d\,. \tag{2.8}$$

The above mentioned anomalies of two linear combinations of U(1)'s are cancelled by a generalized four-dimensional Green-Schwarz mechanism which may be summarized as follows. In type-II string theory there are some closed string "Ramond-Ramond"(RR) modes coupling to the gauge fields. In particular in the D6- and D5-brane models here discussed there are four RR two-form fields B_i with couplings to the $\mathrm{U}(1)_\alpha$ field strengths:

$$\sum_i c_i^\alpha B_i \wedge \mathrm{tr}\,(F^\alpha)\ , \qquad i = 1,2,3,4\,; \qquad \alpha = a,b,c,d \tag{2.9}$$

and in addition there are couplings of the Poincaré dual scalars (representing the same degrees of freedom) η_i of the B_i fields:

$$\sum_i d_i^\beta \eta_i\, \mathrm{tr}\left(F^\beta \wedge F^\beta\right), \tag{2.10}$$

where F^β are the field strengths of any of the gauge groups. The combination of both couplings, by tree-level exchange of the RR-fields, cancels the mixed $\mathrm{U}(1)_\alpha$ anomalies $A_{\alpha\beta}$ with any other group G_β as:

$$A_{\alpha\beta} + \sum_i c_i^\alpha d_i^\beta = 0\,. \tag{2.11}$$

The coefficient c_i^α and d_i^α may be computed explicitly for each given D-brane configuration and we will give later on their values for D6- and D5-brane models. Note that for given α, β, both c_i^α *and* d_i^β have to be non-vanishing for some i, in order to cancel the anomalies.

As remarked in the previous subsection, the couplings in (2.9) give masses to some linear combinations of U(1)'s. Indeed, after a duality transformation the $B \wedge F$ couplings turn into explicit mass terms for the Abelian gauge bosons given by the expression:

$$(M^2)_{\alpha\beta} = g_\alpha g_\beta M_S^2 \sum_{i=1}^{3} c_i^\alpha c_i^\beta \,, \qquad \alpha, \beta = a, b, c, d \,. \tag{2.12}$$

where the sum runs over the massive RR-fields present in the models and where g_α is the coupling of $\mathrm{U}(1)_\alpha$. Here we have normalized to unity the gauge boson kinetic functions.

In principle there could be $4 \times 4 = 16$ c_i^β coefficients. However, it turns out that in the class of D6- and D5-brane models addressed here, there are only seven non-vanishing c_i^β coefficients and only five of them are independent. This is due to the particular structure of the anomalies of this system of four U(1)'s. For both D6- and D5-brane models there are three RR fields B_2^i, $i = 1, 2, 3$ which have the following structure of $B \wedge F^\alpha$ couplings [8, 14]:

$$\begin{aligned}
&B_2^1 \wedge c_1^b \, F^b \,, \\
&B_2^2 \wedge c_2^d \left(-3F^a + F^d \right) , \\
&B_2^3 \wedge \left[c_3^a F^a + c_3^b F^b + \left(\frac{1}{3} c_3^a + c_3^d \right) F^c + c_3^d F^d \right] ,
\end{aligned} \tag{2.13}$$

whereas the fourth antisymmetric field B_2^0 does not couple to any of the U(1)'s. The universality of the form of the anomalous U(1)'s is the cause for the relationship $c_2^a = -3c_2^d$. In addition the equality $c_3^c = (\frac{1}{3} c_3^a + c_3^d)$ is required so that the physical hypercharge combination eq. (2.8) does not mix with B_2^3 and remains massless. We thus see that the most general U(1) mass matrix appearing in this class of models depends on only five free parameters: $c_1^b, c_2^d, c_3^a, c_3^b$ and c_3^d. As mentioned, these five coefficients may be computed for each model. We now provide their value for D6-brane and D5-brane models, respectively.

2.3 D6-brane orientifold models

The structure of these models is discussed in ref. [8]. One considers type-IIA string theory compactified on a six-torus $T^2 \times T^2 \times T^2$. Now we consider D6-branes containing inside Minkowski space and wrapping each of the three remaining dimensions of the branes on a different torus T^2. We denote by $(n_{\alpha i}, m_{\alpha i})$, i=1,2,3, $\alpha = a, b, c, d$ the wrapping numbers[1] of each brane $D6_\alpha$, $n_{\alpha i}(m_{\alpha i})$ being the number of times the brane is wrapping around the x(y)-coordinate of the i-th torus. In this class of models the coefficients c_i^α are given by [8]:

$$c_i^\alpha = N_\alpha n_{\alpha j} \, n_{\alpha k} \, m_{\alpha i} \,; \qquad i \neq j \neq k \neq i \,, \qquad i = 1, 2, 3 \,, \tag{2.14}$$

where N_α is the number of parallel branes of type α. The general set of possible wrapping numbers yielding just the SM fermion spectrum was provided in ref. [8] and is shown

N_α	$(n_{\alpha 1}, m_{\alpha 1})$	$(n_{\alpha 2}, m_{\alpha 2})$	$(n_{\alpha 3}, m_{\alpha 3})$
$N_a = 3$	$(1/\beta_1, 0)$	$(n_{a2}, \epsilon\beta_2)$	$(1/\nu, 1/2)$
$N_b = 2$	$(n_{b1}, -\epsilon\beta_1)$	$(1/\beta_2, 0)$	$(1, 3\nu/2)$
$N_c = 1$	$(n_{c1}, 3\nu\epsilon\beta_1)$	$(1/\beta_2, 0)$	$(0, 1)$
$N_d = 1$	$(1/\beta_1, 0)$	$(n_{d2}, -\beta_2\epsilon/\nu)$	$(1, 3\nu/2)$

Table 3: D6-brane wrapping numbers giving rise to a SM spectrum. The general solutions are parametrized by a phase $\epsilon = \pm 1$, the NS background on the first two tori $\beta_i = 1 - b_i = 1,\ 1/2$, four integers $n_{a2}, n_{b1}, n_{c1}, n_{d2}$ and a parameter $\nu = 1, 1/3$.

in table 3. From table 3 and eq. (2.14) one can compute the relevant c_i^α coefficients:

$$c_1^b = -\frac{2\epsilon\beta_1}{\beta_2}\,; \qquad c_2^d = -\frac{\epsilon\beta_2}{\nu\beta_1}\,; \qquad c_3^a = \frac{3n_{a2}}{2\beta_1}\,;$$
$$c_3^b = \frac{3\nu n_{b1}}{\beta_2}\,; \qquad c_3^c = \frac{n_{c1}}{\beta_2}\,; \qquad c_3^d = \frac{3\nu n_{d2}}{2\beta_1} \tag{2.15}$$

in terms of the free parameters n_{a2}, n_{d2}, n_{b1}, ϵ and β_i. To reduce the number of free parameters in the numerical study of the next sections we will consider a subclass of models (later called class A and B) with the simplest Higgs structure [8], in which the number of free parameters is reduced.

Another interesting subclass of models are those in which for certain choices of the torus moduli all the brane intersections become "approximately" supersymmetric (see section 5.1 in ref. [13])). In that subclass of models one has:

$$\nu = \frac{1}{3}\,; \qquad \epsilon = 1\,;$$
$$n_{a2} = n_{d2}\,; \qquad n_{c1} = \frac{\beta_2}{\beta_1} n_{a2} \tag{2.16}$$

so we are left with n_{a2}, n_{b1} and β_i as free parameters. A specific example of a model with these characteristics (and with $\beta_1 = 1/2$) is presented later in the text, see the third line in table 6.

2.4 D5-brane orientifold models

These models are obtained from type-IIB compactification on an orbifold $T^2 \times T^2 \times (T^2/Z_N)$ (see ref. [14] for details). There are four stacks a,b,c and d of D5-branes which are wrapping cycles on $T^2 \times T^2$ and are located at a Z_N fixed point on T^2/Z_N. The six-dimensional world-volume of the D5-branes includes Minkowski space and the two extra dimensions wrap a different 2-torus. Thus each stack of D5-branes are specified by giving the wrapping numbers $(n_{\alpha i}, m_{\alpha i})$, $i = 1, 2$ as well as the charge with respect to the Z_N symmetry. For general orientifold models of this type one can compute the form of the c_i^α coefficients. In this case there are $2 \times (N-1)$ RR-fields participating in the GS mechanism. In the particularly simple case of a Z_3 orbifold the number of RR antisymmetric B_2^i fields is again four. However, unlike the case of D6-brane, the RR-fields belong to a twisted spectrum.

[1] This notation for $n_{\alpha i}$ is the equivalent of n_α^i used in [8].

Z_3-charge	N_α	$(n_{\alpha 1}, m_{\alpha 1})$	$(n_{\alpha 2}, m_{\alpha 2})$
1	$N_a = 3$	$(-1, \epsilon)$	$(3, -\epsilon/2)$
γ	$N_b = 2$	$(1, 0)$	$(1, -\epsilon/2)$
γ	$N_c = 1$	$(1, 0)$	$(0, \epsilon)$
1	$N_d = 1$	$(2, -3\epsilon)$	$(1, -\epsilon/2))$

Table 4: D5-brane wrapping numbers and Z_3 charges giving rise to a SM spectrum in the example discussed in the text. Here $\gamma = \exp(i2\pi/3)$.

Otherwise, the general structure of the U(1) symmetries is surprisingly analogous to the case of D6-branes. One can find specific examples of Z_N symmetries and wrapping numbers yielding the SM fermion spectrum, although there are no large families of models as in the D6-brane case. The spectrum and U(1) charges are as in table 2. We will provide here just a specific Z_3 example extracted from ref. [14]. In this model the wrapping numbers and Z_3 charges of the four stacks of branes are shown in table 4.

As discussed in ref. [14], in the case of the Z_3 orbifold with the structure of this example one of the four RR-fields does not mix with any U(1) (as in the D6-brane case). The other three have c_i^α, $i = 1, 2, 3$ coefficients given by [14]

$$
\begin{aligned}
c_1^\alpha &= 2C_1 N_\alpha^k \, n_{\alpha 1} \, n_{\alpha 2} \sin(2k\pi/3) \, , \\
c_2^\alpha &= -2C_1 N_\alpha^k \, m_{\alpha 1} \, n_{\alpha 2} \, \cos(2k\pi/3) \, , \\
c_3^\alpha &= -2C_1 N_\alpha^k \, n_{\alpha 1} \, m_{\alpha 2} \, \cos(2k\pi/3) \, ,
\end{aligned} \tag{2.17}
$$

where $C_1 = \sqrt{\sin(2\pi/3)}$. In these equations we have $k = 0$ ($k = 1$) for branes with Z_3-charge equal to 1 ($\exp(i2\pi/3)$), respectively. Thus for the example in table 4 one finds (we take $\epsilon = 1$):

$$
\begin{aligned}
c_1^b &= 2C_1\sqrt{3} \, ; \qquad c_2^a = -18C_1 \, ; \qquad c_2^d = 6C_1 \, ; \\
c_3^a &= -3C_1 \, ; \qquad c_3^b = -C_1 \, ; \qquad c_3^c = C_1 \, ; \qquad c_3^d = 2C_1
\end{aligned} \tag{2.18}
$$

and the rest of the c_i^α vanish. Note that in a specific example like this there is just one free parameter, which is the overall mass scale related to the string scale. We will not give further details of this construction and refer to [14] for further details. We only note that there are Higgs scalars with charges identical to those in the D6-brane models and hence one can make a unified treatment of electroweak symmetry breaking in both classes of models. In this particular example the Higgs sector is formed by one set of fields (H_i, i=1,2) as will be described later in the text, table 5.

2.5 U(1) gauge coupling constant normalization

The gauge couplings of the four U(1)'s have some interesting relationships in the D-brane models of particle physics. Consider in particular a SU(N) non-Abelian group arising from a stack of N parallel branes. Let us normalize as usual the non-Abelian gauge coupling g_N so that the quadratic Casimir in the fundamental is 1/2. Further, the diagonal U(1)

Higgs σ	q_b	q_c	$q'_1 \equiv q_y$	$T_3 = 1/2\,\sigma_z$
h_1	1	-1	$1/2$	$+1/2$
h_2	-1	1	$-1/2$	$-1/2$
H_1	-1	-1	$1/2$	$+1/2$
H_2	1	1	$-1/2$	$-1/2$

Table 5: Higgs fields, their $\mathrm{U}(1)_{b,c}$ charges and weak isospin with σ_z the diagonal Pauli matrix.

Higgs	ν	β_1	β_2	n_{a2}	n_{b1}	n_{c1}	n_{d2}	N_h
$n_H = 1, n_h = 0$	$1/3$	$1/2$	β_2	n_{a2}	-1	1	$\frac{1}{\beta_2} - n_{a2}$	$4\beta_2(1-n_{a2})$
$n_H = 1, n_h = 0$	$1/3$	$1/2$	β_2	n_{a2}	1	-1	$-\frac{1}{\beta_2} - n_{a2}$	$4\beta_2(1-n_{a2})$
$n_H = 0, n_h = 1$	$1/3$	$1/2$	β_2	n_{a2}	1	1	$\frac{1}{\beta_2} - n_{a2}$	$4\beta_2(1-n_{a2}) - 1$
$n_H = 0, n_h = 1$	$1/3$	$1/2$	β_2	n_{a2}	-1	-1	$-\frac{1}{\beta_2} - n_{a2}$	$4\beta_2(1-n_{a2}) + 1$
$n_H = 1, n_h = 1$	1	1	β_2	n_{a2}	0	1	$\frac{1}{3}(\frac{2}{\beta_2} - n_{a2})$	$\beta_2(8 - \frac{4n_{a2}}{3}) - \frac{1}{3}$
$n_H = 1, n_h = 1$	1	1	β_2	n_{a2}	0	-1	$\frac{1}{3}(-\frac{2}{\beta_2} - n_{a2})$	$\beta_2(8 - \frac{4n_{a2}}{3}) + \frac{1}{3}$
$n_H = 1, n_h = 1$	$1/3$	1	β_2	n_{a2}	0	1	$\frac{2}{\beta_2} - n_{a2}$	$\beta_2(8 - 4n_{a2}) - 1$
$n_H = 1, n_h = 1$	$1/3$	1	β_2	n_{a2}	0	-1	$-\frac{2}{\beta_2} - n_{a2}$	$\beta_2(8 - 4n_{a2}) + 1$

Table 6: Families of D6-brane models with minimal Higgs content [8]. The first four lines correspond to models of Class A, the remaining ones to models of Class B. $\beta_2 = 1, 1/2$. The parameters of the models are n_{a2} and g_d/g_c.

field living in the same stack and giving charges ± 1 to the bi-fundamental fermions at the intersections will have gauge coupling $g_1 = g_N/\sqrt{2N}$. For the case of the SM we will thus have that at the string scale:

$$g_a^2 = \frac{g_{QCD}^2}{6}\,; \qquad g_b^2 = \frac{g_L^2}{4}\,, \tag{2.19}$$

where g_L is the $\mathrm{SU}(2)_L$ coupling. In addition, due to eq. (2.8) one has for the hypercharge coupling:[2]

$$\frac{1}{g_Y^2} = \frac{1}{36g_a^2} + \frac{1}{4g_c^2} + \frac{1}{4g_d^2}\,. \tag{2.20}$$

Thus g_a and g_b are determined by eqs. (2.19) whereas g_c and g_d are constrained by eq. (2.20), with the ratio g_d/g_c as a free parameter. In fact if we knew precisely the geometry of the brane configuration (torus moduli and shape) and if the brane configuration was fully factorizable in the different tori, one could in principle compute the gauge coupling constants at the string scale directly, since they are given by the inverse of the volume that the branes are wrapping in the tori (see e.g. refs. [6, 8, 13]). For the analysis to follow in the next sections we will keep the ratio g_d/g_c as a free parameter.[3]

[2]This will be derived in detail later in the text.

[3]In the numerical analysis we will ignore the running between the weak scale and the string scale. This evolution does not affect much the structure of Z' masses and would include model-dependence on the detailed particle content beyond the SM in the region between the weak and the string scales.

We mentioned that the $B \wedge F$ couplings of RR-fields to U(1)'s give masses to these gauge bosons of the order of the string scale M_S. In fact eq. (2.12) assumes a canonical kinetic term for the RR fields which mix with the U(1)'s and give them a mass. However such kinetic terms are field (radii)- dependent (see e.g. ref. [12]) and once one re-defines the fields to canonical kinetic terms, extra volume factors appear in eq. (2.12). In the case of D6-branes one finds that this amounts to the replacement

$$c_i^\alpha \longrightarrow \xi^i c_i^\alpha \, ; \qquad \xi^i = \sqrt{\frac{R_2^i R_1^j R_1^k}{R_1^i R_2^j R_2^k}} \, , \qquad i \neq j \neq k \neq i \, , \tag{2.21}$$

whereas in the case of the Z_3 D5-brane models discussed above the appropriate rescaling factors are

$$\xi^1 = \sqrt{\frac{R_1^1 R_1^2}{R_2^1 R_2^2}} \, ; \qquad \xi^2 = \sqrt{\frac{R_1^1 R_2^2}{R_2^1 R_1^2}} \, ; \qquad \xi^3 = \sqrt{\frac{R_2^1 R_1^2}{R_1^1 R_2^2}} \, . \tag{2.22}$$

Note that these volume factors are also relevant to the question of creating a hierarchy between the string scale M_S (which in the present paper is assumed to be not far above the electroweak scale) and the Planck scale M_p. In the case of the D5-brane models [14] one can take all the radii $R_{1,2}^i$, $i = 1, 2$ of order one (in string units) and the volume of the third (orbifold) torus very large giving rise to the $M_p \gg M_S$ hierarchy in the standard way [31]. If this is assumed then one can set all $\xi^i = 1$ and the U(1) masses will be given by eq. (2.12). The case of D6-brane models is more subtle. As pointed out in [4] in the case of D6-brane models one cannot make some radii $R_{1,2}^i$ very large in order to obtain $M_p \gg M_S$, the reason being that there are no compact directions which are simultaneously transverse to all the SM branes. In this D6-brane case an alternative for understanding the hierarchy could perhaps be the following. The 6-torus could be small while being connected to some very large volume manifold [8]. For example, one can consider a region of the 6-torus away from the D6-branes, cutting a ball and gluing a throat connecting it to a large volume manifold. In this way one would obtain a low string scale model without affecting directly the brane structure. Alternatively it may be that the apparent large value of the four-dimensional Planck mass could be associated with the localization of gravity on the branes, along the lines of [32]. In the numerical analysis to follow we will neglect volume effects and set $\xi^i = 1$. The reader should however remember that when computing bounds on the string scale M_S, the limits apply modulo the mentioned volume factors.

3. Mass eigenstates & eigenvectors for U(1) fields

In this section we outline the approach we use in sections 4 and 5 to analysing the effects of additional U(1) fields in intersecting D-brane models. The approach may be used for other cases as well, for example models with D-branes at singularities [1, 2]. The analysis to follow is applied to generic D6 and D5-brane models and makes little reference to the explicit details of these models. We address the mass eigenvalue problem for the $\mathrm{U}(1)_\alpha$ fields, which for specific D6 and D5-brane models is thoroughly investigated in sections 4 and 5.

In the basis "a, b, c, d" the kinetic terms of the Abelian fields A_α are all diagonal. However, whether anomalous or not, the Abelian fields have mass terms induced not by a

Higgs mechanism, but through their (field strength) couplings to the three RR two-form fields $B_i (i = 1, 2, 3)$. These mass terms are not diagonal, and are given by the 4×4 symmetric mass matrix $M^2_{\alpha\beta}$ of eq. (2.12). An orthogonal transformation (denoted $\mathcal{F}$) is then introduced to "diagonalise" $M^2_{\alpha\beta}$. Therefore

$$A'_i = \sum_{\alpha=a,b,c,d} \mathcal{F}_{i\alpha} A_\alpha \,, \qquad i = 1, 2, 3, 4 \,, \tag{3.1}$$

where the "primed" states correspond to the mass eigenstate basis, and i is fixed ($i = 1, \ldots 4$) to a value which includes the hypercharge ($i = 1$). Since the matrix $\mathcal{F}$ is orthogonal, gauge kinetic terms are not affected by this transformation.

This is what happens when electroweak symmetry breaking effects are neglected. In the presence of the latter, $M^2_{\alpha\beta}$ has additional corrections. The $\mathrm{U}(1)_\alpha$ fields will then acquire a mass from two sources: the string mechanism mentioned before, and the usual Higgs mechanism, with the contribution of the latter suppressed by M_Z^2/M_S^2.

3.1 U(1) masses before electroweak symmetry breaking

In the absence of the electroweak symmetry breaking, the only contribution to the masses of the U(1) gauge bosons is due to $M^2_{\alpha\beta}$. Its eigenvalues $M_i^2 \equiv \lambda_i M_S^2$ are the roots of

$$\det \left[\lambda M_S^2 I_4 - M^2\right] = 0 \,, \tag{3.2}$$

which can be written as

$$\lambda \left(\lambda^3 + c_3 \lambda^2 + c_2 \lambda + c_1\right) = 0 \,, \tag{3.3}$$

with the following coefficients

$$\begin{aligned} c_3 &= -\,\mathrm{Tr}(M^2) < 0 \,, \\ c_2 &= -\frac{1}{2}\left[\mathrm{Tr}(M^4) - (\mathrm{Tr}\, M^2)^2\right] > 0 \,, \\ c_1 &= -\frac{1}{3}\left[\mathrm{Tr}(M^6) - (\mathrm{Tr}\, M^2)^3\right] + \frac{1}{2}\,\mathrm{Tr}(M^2)\left[\mathrm{Tr}(M^4) - (\mathrm{Tr}\, M^2)^2\right] . \end{aligned} \tag{3.4}$$

Since the mass matrix M^2 is symmetric, its eigenvalues are real. The roots of (3.3) have the general form

$$\begin{aligned} \lambda_1 &= 0 \,, \\ \lambda_2 &= -\frac{c_3}{3} + \frac{2}{3}\sqrt{c_3^2 - 3c_2}\,\cos\left(\frac{\phi}{3}\right) , \\ \lambda_3 &= -\frac{c_3}{3} + \frac{2}{3}\sqrt{c_3^2 - 3c_2}\,\cos\left(\frac{\phi}{3} + \frac{2\pi}{3}\right) , \\ \lambda_4 &= -\frac{c_3}{3} + \frac{2}{3}\sqrt{c_3^2 - 3c_2}\,\cos\left(\frac{\phi}{3} + \frac{4\pi}{3}\right) . \end{aligned} \tag{3.5}$$

with the definition

$$\cos\phi = \pm\left[\frac{(2c_3^3 - 9c_2c_3 + 27c_1)^2}{4(c_3^2 - 3c_2)^3}\right]^{1/2} , \tag{3.6}$$

where the "+" ($-$) sign applies if $(2c_3^3 - 9c_2c_3 + 27c_1) < 0$ (> 0). In realistic models $\lambda_1 = 0$ is usually the hypercharge, the massless state before electroweak symmetry breaking. If $c_1 = 0$, a second massless state exists which may correspond to B–L (see ref. [13]). Since $c_3 < 0$ and $c_2 > 0$ there cannot be a third massless state (unless the matrix is identically zero).

For a generic D5-brane model the coefficients c_2^a, c_3^a, c_1^b, c_3^b, c_3^c, c_2^d, c_3^d are non-zero, see (2.18). If in addition $c_2^a = 0$, we obtain the D6-brane models case, see eq. (2.15). In such cases one may show that the coefficient c_1 of (3.4) equals

$$c_1 = -(c_1^b)^2 \left[(c_3^c)^2 \left((c_2^a)^2 + \left(c_2^d\right)^2 \right) + \left(c_2^a c_3^d - c_2^d c_3^a \right)^2 \right] < 0 \tag{3.7}$$

which is thus negative for both D5 and D6-brane models. Taking account of the signs of $c_i (i = 1, 2, 3)$ one can prove that the eigenvalues $\lambda_i > 0 (i = 2, 3, 4)$. This is so because the function $f(\lambda) \equiv (\lambda^3 + c_3\lambda^2 + c_2\lambda + c_1) < 0$ for $\lambda = 0$ and its second derivative $f''(\lambda = 0) = 2c_3 < 0$. Thus λ_i and the (squared) masses $M_i^2 \equiv \lambda_i M_S^2$ are all positive.

After computing the eigenvalues M_i^2, the associated eigenvectors $w_i (i = 1, 2, 3, 4)$ with components $(w_i)_\alpha$ $(\alpha = a, b, c, d)$ may be written as

$$(w_i)_\alpha = - \sum_{\beta = a,b,c} \left(M^2 - \lambda_i M_S^2 I_4 \right)^{-1}_{\alpha\beta} M^2_{\beta d} (w_i)_d \,, \qquad i = 1, \ldots, 4 \,, \qquad \alpha = a, b, c \,. \tag{3.8}$$

The solution can be written more explicitly in terms of the coefficients c_i^α, but the expressions are long and not presented here. As we will see later, there are classes of models[4] for which $\mathrm{U}(1)_b$ field does not mix with the remaining ones (being a mass eigenstate itself), but this depends on the specific structure of the matrix $M^2_{\alpha\beta}$ and will not be addressed here. Finally, the matrix $\mathcal{F}$ of (3.1) is found from eigenvectors w_i (normalised to unity).

3.2 U(1) masses after electroweak symmetry breaking

After electroweak symmetry breaking, the mass matrix $M^2_{\alpha\beta}$ receives corrections due to mixing of initial $\mathrm{U}(1)_\alpha$ with the Higgs sector charged under some of these symmetries and also from the mixing with W_3^μ boson of $\mathrm{SU}(2)_L$ symmetry. These corrections are proportional to $\eta = \langle H \rangle^2 / M_S^2$ where $\langle H \rangle$ stands for the vev in the Higgs sector. The previous mass eigenstates (3.5) will thus have additional corrections of this order with a further (non-zero) mass eigenstate to correspond to Z boson which thus mixes with initial $\mathrm{U}(1)_\alpha$. Taking account of such corrections requires one to "diagonalise" a 5×5 matrix in the basis a, b, c, d *and* W_3^μ. This matrix has the structure $\mathcal{M}^2 = M^2 + \Delta$, where M^2 is just that of (2.12) extended to a 5×5 one by a fifth line and column which have all entries equal to zero. For illustration we present below the electroweak corrections Δ (in

[4]For this see section 4.1.2.

the aforementioned basis) for D6 and D5-brane models[5]

$$\Delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \eta g_b^2 & \eta\delta g_b g_c & 0 & -\frac{1}{2}\eta\delta g_b g_L \\ 0 & \eta\delta g_b g_c & \eta g_c^2 & 0 & -\frac{1}{2}\eta g_c g_L \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2}\eta\delta g_b g_L & -\frac{1}{2}\eta g_c g_L & 0 & \frac{1}{4}\eta g_L^2 \end{pmatrix}, \tag{3.9}$$

where the Higgs sector was assumed to be charged only under $U(1)_b$ and $U(1)_c$ (which is the case in the specific models under study) while δ is just a mixing parameter in this sector. The mass eigenvalue equation of $\mathcal{M}^2$ is now

$$\lambda\left(\lambda^4 + o_3\,\lambda^3 + o_2\,\lambda^2 + o_1\,\lambda + o_0\right) = 0\,, \tag{3.10}$$

with

$$\begin{aligned} o_i &= c_i + \eta s_i + \eta^2\,t_i\,, \qquad i = 1,2,3\,. \\ o_0 &= \eta\,s_0 + \eta^2\,t_0 \end{aligned} \tag{3.11}$$

to be compared to the one in the absence of electroweak symmetry breaking, eq. (3.3). One can compute explicitly the coefficients s_i and t_i which arise from the matrix eq. (3.9) or other similar electroweak corrections.

Equation (3.10) shows that a massless state $\lambda_1 = 0$ exists after electroweak symmetry breaking (photon). The remaining eigenvalues λ_i may be computed as an expansion in the parameter η about their value λ_i in the absence of electroweak effects (given in (3.5)). For example

$$\lambda_i^{\rm EW} = \lambda_i + \eta\lambda_i' + \eta^2\lambda_i'' + \cdots\,, \qquad i = 2,3,4\,. \tag{3.12}$$

Inserting this into (3.10) and taking account of (3.11) one can solve for the coefficients λ_i' and λ_i'' to find the (electroweak) corrected values of the U(1) masses. Similarly, the mass eigenvalue $M_Z^2 = \lambda_5 M_S^2$ corresponding to Z boson may be written as an expansion

$$\lambda_5 = \xi_1\eta\left(1 + \eta\,\xi_{21} + \eta^2\xi_{31} + \cdots\right). \tag{3.13}$$

Using this in (3.10) one computes the coefficients ξ_1, ξ_{21}, ξ_{31} to find

$$\begin{aligned} \xi_1 &= -\frac{s_0}{c_1}\,, \\ \xi_{21} &= \frac{1}{c_1^2}(c_2 s_0 - c_1 s_1) + \frac{t_0}{s_0}\,, \\ \xi_{31} &= \frac{1}{c_1^4}\left[2c_2^2 s_0^2 - c_1 s_0(3c_2 s_1 + c_3 s_0) + c_1^2\left(s_1^2 + s_0 s_2\right) + t_0\left(2c_1^2 c_2 - \frac{s_1 c_1^3}{s_0}\right) - t_1 c_1^3\right]. \end{aligned} \tag{3.14}$$

To leading order in η one should recover the usual mass of Z boson, while corrections of higher order in η are due to mixing between the masses of initial $U(1)_\alpha$ fields and Z boson. Such corrections to the mass of Z boson are of string origin as previously discussed,

[5]The matrix Δ will be derived in detail in section 5. Its exact structure for the general discussion of this section is irrelevant.

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and they will be extensively investigated for specific models in the next two sections. (the eigenvectors after electroweak symmetry breaking may also be computed using this approach, as outlined in appendix A for the eigenvector of Z boson).

4. Analysis of explicit D6 and D5 models before EWS breaking

Following the steps outlined above, we address in this section explicit D-brane models, for which the couplings (2.9) of antisymmetric B-fields to the Abelian gauge bosons are known in detail. This means that the matrix $M^2_{\alpha\beta}$ of (2.12) (essentially its entries c_i^α) is fully known in terms of the (chosen) parameters of the models. We thus consider explicit constructions of D6- and D5-brane models discussed in section 2, in the absence of electroweak symmetry (EWS) breaking effects.

4.1 D6-brane models. Masses of U(1) fields and bounds on M_S

For D6-brane models the coefficients c_i^α in the mass matrix of (2.12) are given in eq. (2.14) and table 3. We analyse a subclass of the D6-brane models, with parameters as presented in table 6 (these models were discussed in ref. [8]). For these models there are two types of Higgs scalars denoted h_i and H_i, respectively, as shown in table 5. For certain choices of integer parameters the number of these Higgs multiplets is minimal, and this defines two classes of models: Class A models which have a single set of Higgs multiplets, either h_i or H_i and Class B models which have both h_i and H_i. Class A models are defined in the first four entries of table 6. Class B models are presented by the last four lines in table 6.

For these specific models we investigate the eigenvalue problem for $M^2_{\alpha\beta}$ and the transformations of couplings and hypercharges of U(1) fields upon "diagonalising" this mass matrix.

4.1.1 Class A models.

The parameters of this class of models are detailed in table 6. For generality we keep $\epsilon, \nu, \beta_1, n_{b1}, n_{c1}$ unassigned, to cover simultaneously all possible cases of this class. We however use the definition of n_{d2} (table 6) ensuring a vanishing mass for the hypercharge state. The ratio g_d/g_c and n_{a2} are chosen as free parameters. The eigenvalues $M_i^2 \equiv \lambda_i M_S^2$ are the roots of

$$\lambda^4 + c_3\lambda^3 + c_2\lambda^2 + c_1\lambda = 0 \tag{4.1}$$

with the notation

$$\begin{aligned}
c_1 &= -\frac{4\epsilon^4 g_b^2 n_{c1}^2}{\beta_2^2\nu^2}\left(g_c^2 g_d^2 + 9g_a^2(g_c^2+g_d^2)\right) < 0\,,\\
c_2 &= \frac{\epsilon^2}{\beta_1^2\beta_2^4\nu^2}\Big\{\beta_2^2 g_b^2\left(9g_a^2+g_d^2\right)\left[4\beta_1^2\beta_2^2\epsilon^2+\nu^2\left(\beta_1^2 n_{a2}^2+9\beta_2^2 n_{b1}^2\right)\right]+\\
&\qquad + n_{c1}^2(9g_a^2\beta_2^4+4\beta_1^4\nu^2 g_b^2)(g_c^2+g_d^2)+\beta_2^4 g_c^2 g_d^2 n_{c1}^2 - 4\beta_1^3\beta_2 g_b^2 g_d^2 n_{a2} n_{c1}\nu^2\Big\} > 0\,,\\
c_3 &= \frac{-1}{4\beta_1^2\beta_2^2\nu^2}\Big\{(9g_a^2+g_d^2)\left(4\beta_2^2\epsilon^2+\nu^2 n_{a2}^2\right)\beta_2^2+\\
&\qquad + 4\beta_1\nu^2\left[4\beta_1^3\epsilon^2 g_b^2 - \beta_2 g_d^2 n_{a2} n_{c1} + \beta_1(g_c^2+g_d^2)n_{c1}^2\right]+36\beta_1^2 g_b^2 n_{b1}^2\nu^4\Big\} < 0\,.
\end{aligned} \tag{4.2}$$

As discussed in the general case, M_i^2 are real ($M^2_{\alpha\beta}$ symmetric) and one may show that $c_1 < 0$, $c_2 > 0$ and $c_3 < 0$, irrespective of the value of parameters present in their definitions above. Thus all M_i^2 are positive. Using eq. (3.5) and coefficients c_i of eqs. (4.2) one finds the values of M_i (λ_i). The eigenvectors may now be computed and are given by:

$$
\begin{aligned}
w_1 &= \frac{1}{|w_1|}\left\{\frac{g_d}{3g_a}, 0, -\frac{g_d}{g_c}, 1\right\}_\alpha, \qquad \alpha = a, b, c, d\,,\\
w_i &= \frac{1}{|w_i|}\left\{w_{ia}, w_{ib}, w_{ic}, 1\right\}, \qquad i = 2, 3, 4\,;
\end{aligned}
\tag{4.3}
$$

with the first three components given by

$$
\begin{aligned}
w_{ia} &= \frac{3\beta_2\, g_a\left(2\beta_2\, g_d^2\, n_{c1}\epsilon^2 - n_{a2}\lambda_i\beta_1\nu^2\right)}{g_d\left[18\beta_2^2 g_a^2 n_{c1}\epsilon^2 + \beta_1\nu^2\lambda_i(n_{a2}\beta_2 - 2\beta_1 n_{c1})\right]},\\
w_{ib} &= -\frac{4\beta_1^2\beta_2^2\,\nu^2\lambda_i^2 + 4\beta_2^2\epsilon^2\left[g_c^2 g_d^2 + 9g_a^2(g_c^2+g_d^2)\right] n_{c1}^2}{6\nu\, n_{b1} g_b g_d\left[18\beta_2^2 g_a^2 n_{c1}\epsilon^2 + \beta_1\nu^2\lambda_i(n_{a2}\beta_2 - 2\beta_1 n_{c1})\right]} +\\
&\quad + \frac{\lambda_i\left[\beta_2^2(9g_a^2+g_d^2)(4\beta_2^2\epsilon^2 + \nu^2 n_{a2}^2) - 4\beta_1\nu^2 n_{c1}(\beta_2 g_d^2 n_{a2} - \beta_1 n_{c1}(g_c^2+g_d^2)\right]}{6\nu\, n_{b1} g_b\, g_d\left[18\beta_2^2 g_a^2 n_{c1}\epsilon^2 + \beta_1\nu^2\lambda_i(n_{a2}\beta_2 - 2\beta_1 n_{c1})\right]},\\
w_{ic} &= \frac{2g_c n_{c1}\left[\beta_2^2\epsilon^2(9g_a^2+g_d^2) - \beta_1^2\nu^2\lambda_i\right]}{g_d\left[18\beta_2^2 g_a^2 n_{c1}\epsilon^2 + \beta_1\nu^2\lambda_i(n_{a2}\beta_2 - 2\beta_1 n_{c1})\right]}
\end{aligned}
\tag{4.4}
$$

and where for each eigenvector w_i (i=2,3,4) the appropriate eigenvalue λ_i as given by eqs. (3.5) is substituted. From the above equations we note that the anomalous $\mathrm{U}(1)_b$ state mixes with the other states upon diagonalising the mass matrix (which is unlike Class B models to follow). The eigenvectors above will be used in analysing the "amount" of each initial $\mathrm{U}(1)_\alpha$ present in the final mass eigenstates $M_{2,3,4}$.

We have so far presented the eigenvalues and eigenvectors of $M^2_{\alpha\beta}$ in terms of the parameters of this class of models. Further, the couplings of the new (mass eigenstates) U(1) fields also change upon going to the new basis. The new couplings g_i' can be expressed in terms of the "old" couplings g_α, and similar relations exist for the transformation of the charges $q_\alpha \to q_i'$. First, the eigenvectors w_i of eqs. (4.3) define the matrix $\mathcal{F}_{i\alpha} = (w_i)_\alpha$ introduced in eq. (3.1), which respects the conditions $\mathcal{F}\mathcal{F}^T = \mathcal{F}^T\mathcal{F} = 1$ and $\mathcal{F}M^2\mathcal{F}^T = \delta_{ij}M_i^2$. The U(1) charges of the re-defined Abelian fields (mass eigenstates) can then be computed by using the invariance of the following term which is part of the covariant derivative

$$
\sum_{\alpha=a,b,c,d} q_\alpha g_\alpha A_\alpha
\tag{4.5}
$$

under changing the basis from "a, b, c, d" to the "primed" basis. The properties of $\mathcal{F}$ then give the transformation of U(1) charges:

$$
q_i' = \sum_{\alpha=a,b,c,d} U_{i\alpha} q_\alpha\,, \qquad U_{i\alpha} \equiv \frac{g_\alpha}{g_i'}\mathcal{F}_{i\alpha}\,, \qquad i = 1, 2, 3, 4\,.
\tag{4.6}
$$

From this equation we notice that the new hypercharges q_i' of associated fields depend in general (see definition of $U_{i\alpha}$) on the couplings of the model. For the massless state (hypercharge) we find

$$q_1' = \frac{g_d}{3g_1'|w_1|}\,(q_a - 3q_c + 3q_d)\,, \qquad (q_1' \equiv q_y,\; g_1' \equiv g_y)\,. \tag{4.7}$$

With $q_y = 1/6(q_a - 3q_c + 3q_d)$ see table 2, we find the hypercharge coupling is given by

$$\frac{1}{g_1'^2} = \frac{|w_1|^2}{4g_d^2} = \frac{1}{36}\frac{1}{g_a^2} + \frac{1}{4}\frac{1}{g_c^2} + \frac{1}{4}\frac{1}{g_d^2} \tag{4.8}$$

as anticipated in section 2.5. More generally, the orthogonality of the matrix $\mathcal{F}$ gives the following relation between the "old" and "new" gauge couplings in terms of the matrix $U_{i\alpha}$

$$\frac{\delta_{ij}}{g_i' g_j'} = \sum_{\alpha=a,b,c,d} \frac{U_{i\alpha}U_{j\alpha}}{g_\alpha^2}\,. \tag{4.9}$$

We thus identified the new basis, the mass eigenvalues, and the transition between the old and new couplings and hypercharges of the system of U(1)'s. With these analytical results, we can compute (lower) bounds on the value of the string scale M_S in models of Class A. This is done by analysing the dependence of the eigenvalues eqs. (3.5) with coefficients (4.2) on the parameters of the models.

A thorough analysis of all Class A models parametrised in table 6 would require a separate investigation of each of these models. Although we do not discuss separately each model, the results are generic for this class. As an example we present in figure 2 the dependence of the eigenvalues M_2, M_3 and M_4 in function of n_{a2} and g_d/g_c for[6] a Class A model of table 6 defined by $\beta_1 = 1/2$, $\nu = 1/3$, $\beta_2 = 1$, $n_{c1} = 1$, $n_{b1} = -1$. Figure 2 shows that the eigenvalue M_2 is larger than the string scale ($M_2 \geq 8M_S$). However, for M_4 we have $1.6M_S \leq M_4 \geq 0.4M_S$ but may become smaller for large n_{a2}. Finally M_3 has a mass between $0.15M_S \leq M_3 \leq 0.32\,M_S$ and hence may be small compared to the string scale, particularly for large values of n_{a2}.

One can draw the generic conclusion that the masses of the additional U(1) fields are within a factor of $\mathcal{O}(10)$ from the string scale, either larger or smaller. Given the current generic bounds on the masses of additional U(1) bosons in the region of 500–800 GeV [17], and with the lightest eigenstate satisfying $M_3 \leq 0.32M_S$, one concludes that the string scale must obey $M_S \geq 1.5$–2.4 TeV. For $M_3 \approx 0.15M_S$, $M_S \approx 3$–4.8 TeV. These lower bounds on M_S will increase with n_{a2}, when M_3 decreases. An additional bound on M_S will be derived in section 5 from taking account of the electroweak symmetry breaking effects. The final lower bound on M_S will be provided by the value which satisfies both of the above constraints.

It is interesting to know which of the initial $\mathrm{U}(1)_\alpha$ ($\alpha = a,b,c,d$) dominates in the final mass eigenstates, $M_{2,3,4}$. This depends in general on the parameters of the models of Class A, as it may be seen from the eigenvectors of eq. (4.4). It turns out however that the result is not very sensitive to n_{a2}. The only exception is w_{ib} (i.e. $\mathrm{U}(1)_b$ presence) which has a more significant spreading with respect to n_{a2} for fixed ratio g_d/g_c than the remaining $w_{i\alpha}$. This

[6] All plots of figure 2 take into account the correlation (4.8) of the couplings of additional U(1)'s such as the hypercharge coupling is fixed to the experimental value at the electroweak scale.

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dependence is taken into account for this discussion. The conclusions will then apply generically to every model of Class A of table 6 since the additional dependence of the eigenvector components (mixing) on β_i, n_{c1}, n_{b1} is rather mild as may be seen by examining separately every model of Class A. Therefore the only significant change of the amount of $\mathrm{U}(1)_\alpha$ each mass eigenstate has is due to changes of the parameter g_d/g_c and this is examined below.

Consider first the mass eigenstate M_2 (figure 2). M_2 is for $g_d/g_c < 1$ a mixture of $\mathrm{U}(1)_a$ and $\mathrm{U}(1)_d$ where $\mathrm{U}(1)_a$ dominates. The same holds true for the case $g_d/g_c > 1$, with $\mathrm{U}(1)_a$ to dominate more for g_d/g_c small, of of order one, and less when this ratio increases (top curve) in figure 2B. In this last case the amount of initial $\mathrm{U}(1)_a$ and $\mathrm{U}(1)_d$ is comparable. For M_3 mass eigenstate, for $g_d/g_c < 1$ each $\mathrm{U}(1)_b$, $\mathrm{U}(1)_c$ $\mathrm{U}(1)_d$ states contribute with similar/comparable amount, with $\mathrm{U}(1)_b$ and $\mathrm{U}(1)_c$ to dominate for smaller g_d/g_c, top curve in figure 2C. For $g_d/g_c > 1$, $\mathrm{U}(1)_b$, $\mathrm{U}(1)_c$ and $\mathrm{U}(1)_d$ contribute significantly for lower curves, and $\mathrm{U}(1)_b$ dominates for $g_d/g_c \gg 1$, see figure 2D. Finally, for M_4 mass eigenstate, the lower curves in figure 2E have a $\mathrm{U}(1)_b$ dominating component and additional comparable amount of $\mathrm{U}(1)_c$ and $\mathrm{U}(1)_d$. While decreasing g_d/g_c, top curve, $\mathrm{U}(1)_b$ dominates more strongly, with additional amount of $\mathrm{U}(1)_a$. For the case of $g_d/g_c \gg 1$, figure 2F $\mathrm{U}(1)_b$ again dominates for lower curves, while top curves are an equal mixture of $\mathrm{U}(1)_b$ and $\mathrm{U}(1)_d$ and a slightly larger $\mathrm{U}(1)_a$ component.

To conclude, the anomalous component $\mathrm{U}(1)_b$ is manifest mostly in M_3 and M_4, while $\mathrm{U}(1)_a$ dominates in M_2 case, with $\mathrm{U}(1)_d$ manifest significantly in all lower curves ($g_d \approx g_c$) in figure 2. Generically, the presence of $\mathrm{U}(1)_a$ favours larger (than the string scale) mass eigenvalues, and $\mathrm{U}(1)_b$ together (to a lesser extent) with $\mathrm{U}(1)_c$ and $\mathrm{U}(1)_d$ is more present in states of mass smaller than the string scale.

4.1.2 Class B models.

The parameters of these models are detailed in table 6. In this case, $n_{b1} = 0$. One also has $\epsilon = \pm 1$, $\nu = 1, 1/3$, $\beta_1 = 1$, $n_{c1} = \pm 1$, $\beta_2 = 1, 1/2$, although we keep all these unassigned, to analyse simultaneously all possible combinations. The free parameters of the model are again chosen as g_d/g_c and n_{a2}. Compared to the general case, the mass matrix $M^2_{\alpha\beta}$ is significantly simpler (since $n_{b1} = 0$). Its eigenvalues M_i^2 are

$$\begin{aligned} M_1^2 &= 0\,, \qquad M_2^2 = (x+y) M_S^2\,, \\ M_3^2 &= \left(\frac{2\beta_1}{\beta_2}\right)^2 g_b^2 \epsilon^2 M_S^2\,, \qquad M_4^2 = (x-y)\, M_S^2 \end{aligned} \tag{4.10}$$

with the notation

$$\begin{aligned} x &= \frac{1}{8\beta_1^2\beta_2^2\nu^2} \left\{ \left[9g_a^2 + g_d^2\right] \left[4\beta_2^4\epsilon^2 + \nu^2\beta_2^2 n_{a2}^2\right] + 4\beta_1 n_{c1}\nu^2 \left[\beta_1(g_c^2+g_d^2)n_{c1} - \beta_2 g_d^2 n_{a2}\right] \right\}, \\ y &= \left\{ x^2 - \frac{\epsilon^2 n_{c1}^2}{\beta_1^2\nu^2} \left[g_c^2 g_d^2 + 9 g_a^2 (g_c^2 + g_d^2)\right] \right\}^{1/2}. \end{aligned} \tag{4.11}$$

The massless eigenvalue corresponds to the observed hypercharge. An interesting consequence of (4.10) and (4.11) is that one may in principle have a very light state M_4, without

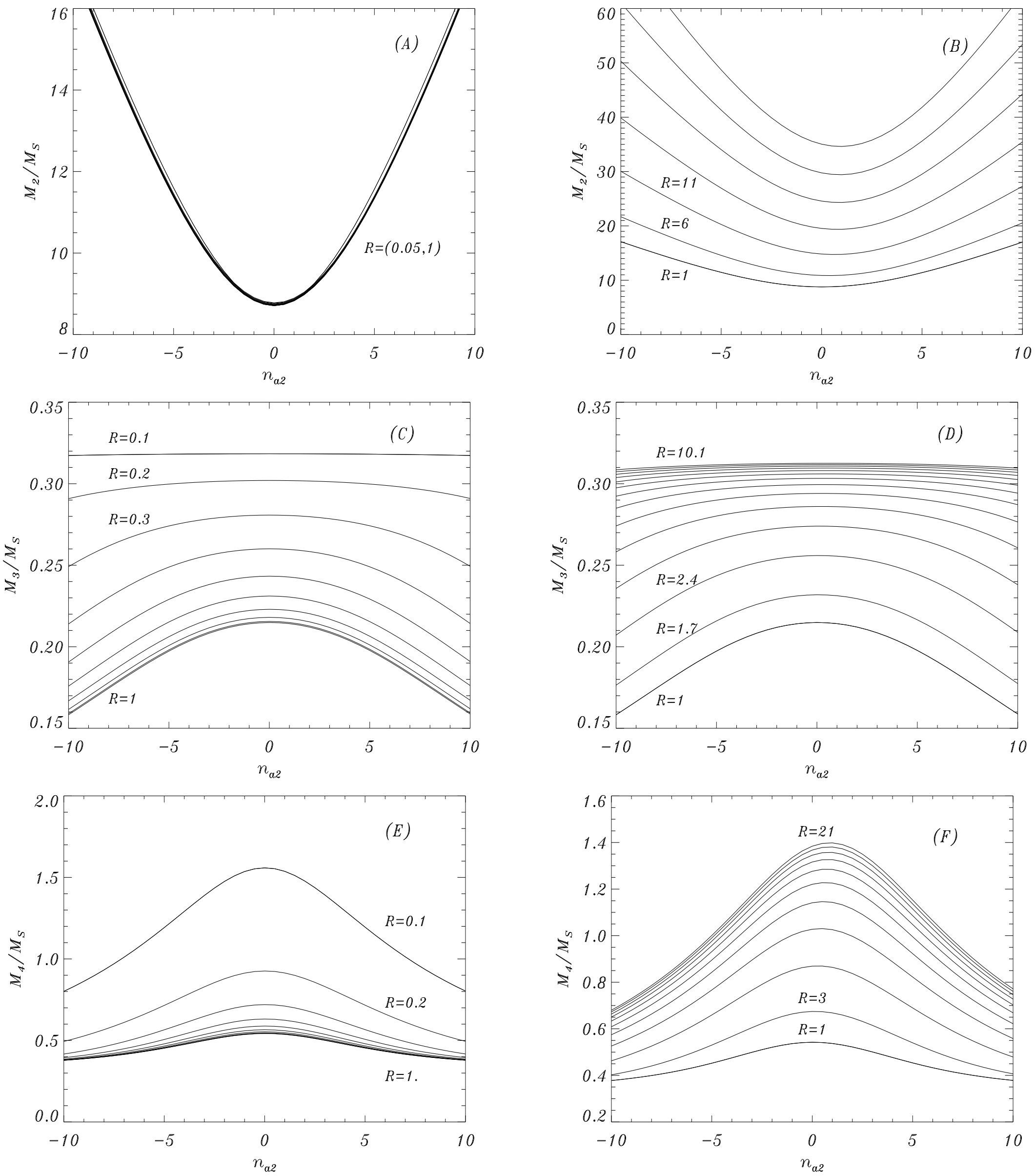

Figure 2: D6-brane models: Class A models of table 6 (with $\beta_1 = 1/2$, $\beta_2 = 1$, $\nu = 1/3$, $n_{c1} = 1$, $n_{b1} = -1$). $M_{2,3,4}$ (string units) in function of n_{a2} for fixed $\mathcal{R} \equiv g_d/g_c$. (A): M_2 for $0.05 \leq \mathcal{R} \leq 1$ has small sensitivity to $\mathcal{R}$. (B): M_2 with $1 \leq \mathcal{R} \leq 26$, $\mathcal{R}$ increasing *upwards*, step 5. (C): M_3 with $0.1 \leq \mathcal{R} \leq 1$ increasing *downwards*, step 0.1. (D): M_3 with $1 \leq \mathcal{R} \leq 10.1$ increasing *upwards*, step 0.7. (E): M_4 with $0.1 \leq \mathcal{R} \leq 1.$ increasing *downwards* step 0.1. (F): M_4 with $1 \leq \mathcal{R} \leq 21$ increasing *upwards* step 2. Electroweak corrections exist for these masses, but they are significantly suppressed by M_Z^2/M_S^2 and are not included here.

the need for a Higgs mechanism. In fact

$$M_2 M_4 = \frac{|\epsilon n_{c1}|}{\beta_1 \nu} \left[g_c^2 g_d^2 + 9 g_a^2 \left(g_c^2 + g_d^2 \right) \right]^{1/2} M_S^2 \,. \tag{4.12}$$

With couplings of fixed value, M_4 may be significantly small(er than the string scale) if

M_2 is made large enough (larger than the string scale) by a large choice for the integer (wrapping number) n_{a2}.

The (normalised) orthogonal eigenvectors associated with the eigenvalues of eq. (4.10) are

$$\begin{aligned} w_1 &= \frac{1}{|w_1|}\left\{\frac{g_d}{3g_a}, 0, -\frac{g_d}{g_c}, 1\right\}_\alpha, \qquad \alpha = a, b, c, d, \\ w_2 &= \frac{1}{|w_2|}\left\{\frac{3g_a}{g_d}\frac{1}{2\omega_1}\left(\omega_2 - 8\beta_1^2\beta_2^2\nu^2 y\right), 0, \frac{g_c}{g_d}\frac{1}{2\omega_1}\left(\omega_3 - 8\beta_1^2\beta_2^2\nu^2 y\right), 1\right\}_\alpha, \\ w_3 &= \{0, 1, 0, 0\}_\alpha, \\ w_4 &= \frac{1}{|w_4|}\left\{\frac{3g_a}{g_d}\frac{1}{2\omega_1}\left(\omega_2 + 8\beta_1^2\beta_2^2\nu^2 y\right), 0, \frac{g_c}{g_d}\frac{1}{2\omega_1}\left(\omega_3 + 8\beta_1^2\beta_2^2\nu^2 y\right), 1\right\}_\alpha \end{aligned} \tag{4.13}$$

with the norm of the vector

$$|w_i| = \left[\sum_\alpha (w_i)_\alpha^2\right]^{1/2} \tag{4.14}$$

and with

$$\begin{aligned} \omega_1 &= 36\beta_2^4\epsilon^2 g_a^2 + \nu^2\left(\beta_2 n_{a2} - 2\beta_1 n_{c1}\right)\left(9\beta_2 g_a^2 n_{a2} + 2\beta_1 n_{c1} g_c^2\right), \\ \omega_2 &= -\left\{\beta_2^2\left(9g_a^2 - g_d^2\right)\left(4\beta_2^2\epsilon^2 + \nu^2 n_{a2}^2\right) + 4\nu^2\beta_1 n_{c1}(g_c^2 + g_d^2)\left(n_{a2}\beta_2 - \beta_1 n_{c1}\right)\right\}, \\ \omega_3 &= \left\{\beta_2\left(9g_a^2 + g_d^2\right)\left(4\beta_2^3\epsilon^2 + \nu^2\beta_2 n_{a2}^2 - 4\nu^2\beta_1 n_{a2} n_{c1}\right) + 4\nu^2\beta_1^2\left(g_d^2 - g_c^2\right) n_{c1}^2\right\}. \end{aligned} \tag{4.15}$$

The first eigenvector in eq. (4.13) is that corresponding to the hypercharge.

It is interesting to notice that unlike Class A models, $\mathrm{U}(1)_b$ (anomalous) does not mix with the rest of the Abelian fields and remains massive. Of the remaining eigenvectors (w_2, w_4), one linear combination is anomalous and one is anomaly free. The eigenvectors $w_{2,3,4}$ can be used as in Class A to investigate which original $\mathrm{U}(1)_\alpha$ dominates in every mass eigenstate (this is done in the last part of this section).

The eigenvectors w_i define the matrix $\mathcal{F}_{i\alpha} = (w_i)_\alpha$, with $\mathcal{F}\mathcal{F}^T = \mathcal{F}^T\mathcal{F} = 1$ and with $\mathcal{F}M^2\mathcal{F}^T = \delta_{ij}M_i^2$ where M^2 is the initial mass matrix, eq. (2.12). The new U(1) charges, including hypercharge (and their normalisation) of the re-defined (mass eigenstates) Abelian fields can be computed as in Class A. We find again ($g_1' \equiv g_y$)

$$\frac{1}{g_1'^2} = \frac{|w_1|^2}{4g_d^2} = \frac{1}{36}\frac{1}{g_a^2} + \frac{1}{4}\frac{1}{g_c^2} + \frac{1}{4}\frac{1}{g_d^2} \tag{4.16}$$

with similar relations to hold for the remaining g_i' and q_i'. Using (4.16) we can also re-write (4.12) in terms of the chosen parameter, the ratio $\mathcal{R} \equiv g_d/g_c$

$$M_2 M_4 = \frac{|\epsilon n_{c1}|}{\beta_1\nu}\frac{54 g_a^3 M_S^2}{g_y(36 g_a^2/g_y^2 - 1)}\left[\mathcal{R} + \frac{1}{\mathcal{R}}\right], \tag{4.17}$$

where g_a is related to QCD coupling, (2.19). Thus the string scale is approximately the geometric mean of M_2 and M_4. This also shows that at fixed g_d/g_c, M_4 may become very

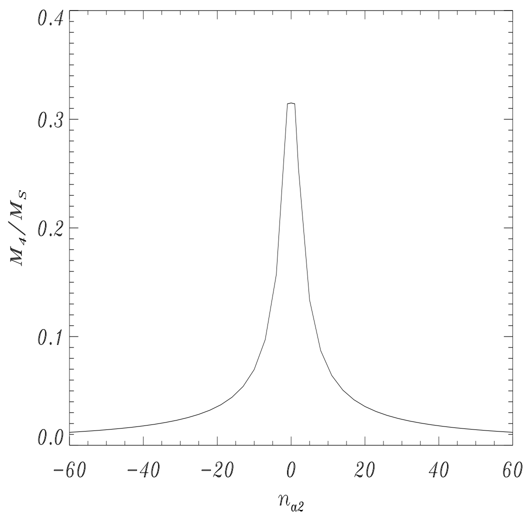

Figure 3: Class B: The curve of minimal values of M_4 (string units) in function of n_{a2}, is realised for $g_d/g_c = 1$. M_4 decreases for larger integer $n_{a2} \approx 60$ to $1/100 M_S$. The mass of this state has a stringy origin rather than a standard Higgs mechanism. It has corrections from the usual Higgs mechanism, suppressed by the ratio M_Z^2/M_S^2. Similar state exists in Class A models, see lower curves in figure 2(C,D,E,F) at large n_{a2}.

light, even of the order of M_Z scale for (fixed) string scale $M_S \approx \mathcal{O}(1\text{–}10\,\text{TeV})$ (see figure 3), without a Higgs mechanism. (This happens if n_{a2} (i.e. M_2) is made large enough). This is an interesting mechanism, regardless of the phenomenological viability of such a light boson. Alternatively, its (light) mass can imply stringent bounds on the string scale by the requirement this light state be massive enough to avoid current experimental bounds.

As mentioned, the $\mathrm{U}(1)_b$ gauge boson with eigenvector w_3 (mass eigenstate M_3) does not mix with the others and one can check that is lighter than M_S, see eq. (4.10). From this eq. for $\beta_1 = \beta_2 = 1$, together with $M_3 > 500\text{–}800\,\text{GeV}$ one finds $M_S > (500\text{–}800)/g_L$ i.e. lower bound of $M_S \approx 800\text{–}1350\,\text{GeV}$. Additional higher (lower) bounds on M_S are provided from the analysis of the remaining mass eigenstates.

Further numerical results for Class B models are presented in figure 4. These figures, (although somewhat generic for all models of this class) correspond to the first example of class B in table 6 (fifth row) defined by $\beta_1 = 1$, $\nu = 1$, $\beta_2 = 1$, $n_{c1} = 1$. The figures present the dependence of the eigenvalues M_4 and M_2 in string units in function of n_{a2} and g_d/g_c. Generically, the masses are within a factor of $\mathcal{O}(10)$ from the string scale, either larger or smaller. With bounds on the gauge bosons of 500–800 GeV, this implies $M_S > 5\text{–}8\,\text{TeV}$. However, for large n_{a2}, the bound on M_S can further increase. (This is the case of figure 3 showing the value of M_4/M_S in function of n_{a2} for $g_d = g_c$, yielding the minimal values for M_4/M_S which together with the constraint $M_4 > 500\text{–}800\,\text{GeV}$, give the highest (lower) bound for M_S). Electroweak corrections to $M_{2,3,4}$ are suppressed by M_Z^2/M_S^2 and small, even for low string scale. Additional bounds on the string scale can be achieved by constraining the correction to M_Z due to new physics (U(1)'s) effects be within the current experimental error on the ρ parameter, and this is the purpose of section 5.

Finally, we analyse the amount of mixing of the initial $\mathrm{U}(1)_\alpha$ ($\alpha = a, b, c, d$), which is encoded by the eigenvector components in (4.13). As for Class A, we do not address every specific model of Class B, but rather consider the pattern that emerges after investigating each of them separately. For class B models we thus address the "amount" $w_{i\alpha}$ ($\alpha = a, c, d$) of original $\mathrm{U}(1)_\alpha$ present in each mass eigenstate other than the $\mathrm{U}(1)_b$ which does not mix. For the hypercharge state, the amount of initial $\mathrm{U}(1)_\alpha$ is somewhat clear, see $w_{1\alpha}$, and only depends on the parameter g_d/g_c. For the remaining $w_{2\alpha}$ and $w_{4\alpha}$, their dependence on n_{a2} for fixed ratio g_d/g_c (with correlation (4.16) to *fixed* g_y included) is stronger than in Class A models. Still, one can say that in M_4 of figure $4E$ upper curves are dominated by $\mathrm{U}(1)_a$ and $\mathrm{U}(1)_c$ while lower curves correspond to states with contributions from $\mathrm{U}(1)_c$, $\mathrm{U}(1)_d$ and $\mathrm{U}(1)_a$. If $g_d/g_c > 1$ figure $4F$, lower curves correspond to $\mathrm{U}(1)_c$, $\mathrm{U}(1)_d$ and $\mathrm{U}(1)_a$, while upper curves are dominated by $\mathrm{U}(1)_a$ and with a less but comparable "amount" of $\mathrm{U}(1)_d$. The cases of figures $4E$ and $4F$ have a correspondent in Class A models in figures $2E$ and $2F$ as far as the size of the mass is concerned. For the plots in figures $4A$ and $4B$ the situation is to some extent similar to the case of M_2 of figures $2A$ and $2B$. Lower curves for M_2 with $g_d/g_c \leq 1$ are dominated by $\mathrm{U}(1)_a$ and $\mathrm{U}(1)_d$ (to a less extent) while upper curves by $\mathrm{U}(1)_{a,c,d}$ in a comparable "amount". In figure $4B$ lower curves are dominated by $\mathrm{U}(1)_a$ (and some less amount of $\mathrm{U}(1)_d$) while the upper curves contain a comparable amount of $\mathrm{U}(1)_{a,d}$.

As in Class A models, $\mathrm{U}(1)_a$ is mostly present in cases with masses larger than the string scale. $\mathrm{U}(1)_d$ is manifest in all lower curves with $g_d \approx g_c$ and $\mathrm{U}(1)_b$ is present unmixed, as a mass eigenstate with M_3 smaller than the string scale. The analysis shows patterns similar to Class A models.

4.2 D5-brane models. masses of U(1) fields and bounds on M_S

Explicit D5-brane models that we briefly address in this section are defined by the mass matrix of eq. (2.12), with coefficients c_i^α presented in eq. (2.17). In this case an analysis similar to that for D6-brane models applies. In the following we choose the simple example of eqs. (2.18) which is rather representative of the SM's considered in ref. [14] and has only one parameter, the ratio g_d/g_c. This allows us to make more definite predictions for the value of the string scale or boson masses. The mass eigenvalues are as usual the roots of the equation

$$\lambda^4 + c_3\lambda^3 + c_2\lambda^2 + c_1\lambda = 0 \tag{4.18}$$

with coefficients

$$\begin{aligned} c_1 &= -162\sqrt{3}\, g_b^2 \left[g_c^2\, g_d^2 + 9g_a^2 \left(g_c^2 + g_d^2\right)\right] < 0\,, \\ c_2 &= 9\left[3\, g_c^2 g_d^2 + g_b^2\left(g_c^2 + 43 g_d^2\right) + 9g_a^2\left(40\, g_b^2 + 3\left(g_c^2 + g_d^2\right)\right)\right] > 0\,, \\ c_3 &= -\sqrt{3}\left(333\, g_a^2 + 13\, g_b^2 + g_c^2 + 40\, g_d^2\right)/2 < 0\,. \end{aligned} \tag{4.19}$$

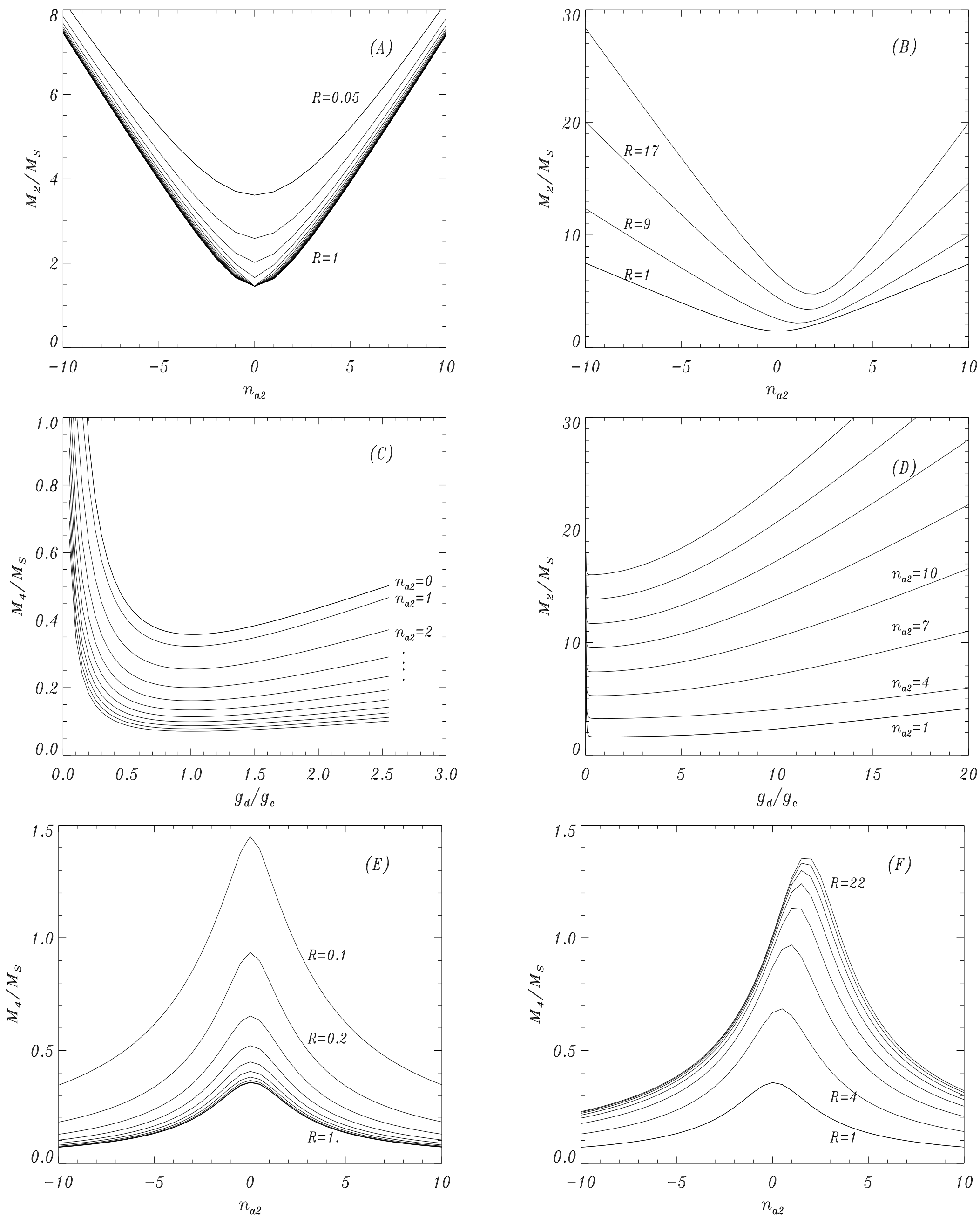

Figure 4: D6-brane models: Class B models of table 6 (with $\beta_1 = \beta_2 = 1$, $\nu = 1$, $n_{c1} = 1$). $M_{2,4}$ (string units), in function of n_{a2} for fixed $\mathcal{R} \equiv g_d/g_c$ (A, B, E, F), or in function of $\mathcal{R}$ for fixed n_{a2} (*C,D*). (A): M_2 for $0.05 \leq \mathcal{R} \leq 1$ increasing *downwards* step 0.02. (B): M_2 for $1 \leq \mathcal{R} \leq 25$ increasing *upwards* step 8. (C): M_4 in function of $\mathcal{R}$, n_{a2} increases *downwards* step 1. (D): M_2 in function of $\mathcal{R}$ for n_{a2} increasing *upwards* step 3. (E): M_4 for $0.1 \leq \mathcal{R} \leq 1$ increasing *downwards* step 0.1. (F): M_4 for $1 \leq \mathcal{R} \leq 22$ increasing *upwards* step 3.

With these coefficients and (3.5) one finds the (real, positive) eigenvalues for this case. One root is again massless, $\lambda_1 = 0$ corresponding to the hypercharge field $\mathrm{U}(1)_Y$. Its associate eigenvector is identical to the corresponding one of the D6-brane models (Class

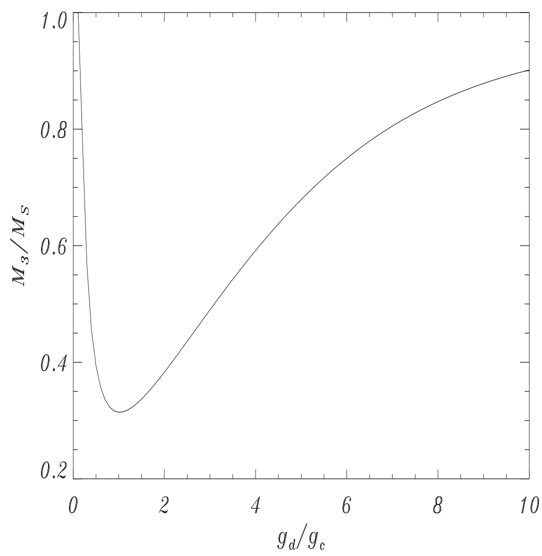

Figure 5: The lightest mass M_3 (string units) of the D5-brane model addressed in section 4.2 in function of g_d/g_c is reached when $g_d \approx g_c$. The remaining (non-vanishing) mass eigenvalues (not shown) are $M_2 \approx (8\text{–}13.5) \times M_S$ and $M_4 \approx M_S$ (for same range of g_d/g_c).

A and B) and leads to the same value for the hypercharge coupling g_y in terms of g_α, ($\alpha = a, \ldots, d$). Hence

$$\frac{1}{g_y^2} = \frac{1}{36}\frac{1}{g_a^2} + \frac{1}{4}\frac{1}{g_c^2} + \frac{1}{4}\frac{1}{g_d^2}\,. \tag{4.20}$$

This value of g_y is compatible with the hypercharges of the SM fields as given in table 2. The constraint (4.20) is again used for evaluating the mass spectrum of the U(1) fields. Here we only discuss the lightest of these states which is $M_3 = \sqrt{\lambda_3} M_S$ in terms of the ratio g_d/g_c. From figure 5 we note the smallest value of M_3 (in string units) exists when the couplings g_d and g_c have close values, and is $M_3 \approx 0.3 M_S$. This value has further corrections from electroweak symmetry breaking, but these are significantly suppressed (by the factor M_Z^2/M_S^2) even for a low string scale and are neglected in this discussion. The remaining mass eigenvalues (not presented) have the values: $M_2 \approx 8 M_S$ for $g_d \approx 0.1 g_c$ with mild variation (increase) up to $M_2 \approx 13.5 M_S$ for $g_d = 10 g_c$. Also $M_4 \approx M_S$ for most values of g_d/g_c between (0.05,20).

Generic constraints on the masses of additional bosons of about 500–800 GeV give the strongest constraints on the value of M_S when they are applied to the lightest (in string units) state M_3 rather than to (larger) $M_{2,4}$. Taking into account the minimal value for $M_3 \approx 0.3 M_S$, one predicts $M_S \geq 1.5\text{–}2.4\,\text{TeV}$. This lower bound on M_S is rather similar (although somewhat smaller) to the previous case of D6-brane models that we addressed. The effects of electroweak symmetry breaking in this D5 model and consequences for M_S values are addressed in section 5.2.

5. Constraints on explicit D6 and D5 models from the ρ-parameter

We have so far investigated the masses of the additional U(1) fields in the absence of electroweak symmetry breaking. At low energies, the effects of electroweak scale physics (EW) become important and the action term below due to (2.12) receives further corrections. Such corrections induced onto the masses of the extra U(1)'s of the previous section may be computed using the procedure outlined in section 3.2, eq. (3.12), as a perturbative expansion in η about their values in the absence of EWS breaking. Since these effects are relatively small, they will not be addressed. In this section we instead compute the corrections induced on the mass of Z boson by the presence of the additional U(1)'s, as outlined in section 3.2. Such corrections, induced by the mixing of the Higgs state with some of the initial U(1)'s (massive) are of stringy origin since the masses of the additional U(1) are themselves of this nature. Therefore the corrections to M_Z we compute are *not* due to a Higgs mechanism. Regardless of the phenomenological viability of our models, we find this mechanism very interesting and worth further attention. Finally, we use the corrections to M_Z that we compute for D6 (Class A and B) and D5-brane models to impose (lower) bounds on the value of M_S, by making use of the current constraints on the value of ρ parameter.

We start with the following term in the action

$$\mathcal{L}_1 = \frac{1}{2}\sum_{\alpha,\beta} M^2_{\alpha\beta} A_\alpha A_\beta \equiv \frac{1}{2}\sum_{\alpha,\beta}\left[g_\alpha g_\beta M_S^2 \sum_{i=1}^{3} c_i^\alpha c_i^\beta\right] A_\alpha A_\beta\,, \qquad \alpha,\beta = a,b,c,d\,. \tag{5.1}$$

Including electroweak symmetry breaking effects on the masses of U(1) fields requires a careful examination of the Higgs sector. We remind the reader that the Higgs sector is charged under both $\mathrm{U}(1)_b$ and $\mathrm{U}(1)_c$, as presented in table 5, thus A_α ($\alpha = b, c$) fields will have couplings at the electroweak scale to $\sigma = H_i$ or h_i and also to W_3^μ. All the couplings of $\mathrm{U}(1)_\alpha$ fields can be read off from the following additional term in the lagrangian

$$\begin{aligned}\mathcal{L}_2 &= (D^\mu\sigma)^\dagger(D_\mu\sigma)\,, \qquad\qquad (5.2)\\ D^\mu\sigma &= \left[\partial^\mu + i g_L \vec{T}.\vec{W}^\mu + i\sum_{\alpha=a,b,c,d} g_\alpha q_\alpha A_\alpha^\mu\right]\sigma = \left[\partial^\mu + i g_L \vec{T}.\vec{W}^\mu + i\sum_{i=1}^{4} g_i' q_i' A_i'^{\mu}\right]\sigma\,.\end{aligned}$$

In the last step, following the previous approach, we made explicit the change of basis performed in previous section. The field σ stands for any of the Higgs fields in table 5. The above equation also shows the relative normalisation of the couplings (also $g_L = 2g_b$) in the $\mathrm{SU}(2)_L$ and U(1) sectors, in agreement with $\mathrm{U}(1)_\alpha$ charge normalisations/assignments in tables 2 and 5.

The mass eigenvalue problem for U(1) fields with electroweak symmetry breaking reduces to diagonalising the 5×5 mass matrix $\mathcal{M}^2_{\gamma\gamma'}$ with $\gamma,\gamma' = \{a,b,c,d,e\}$ which we present below in the basis of $\mathrm{U}(1)_\alpha$ ($\alpha = a,\dots,d$) and W_3^μ. We denote by $v_i/\sqrt{2}$ (i=1,2) the v.e.v. of a Higgs field σ_i with $\sigma_i = H_i$ (Class A, $n_H = 1, n_h = 0$) or $\sigma_i = h_i$ (Class A, $n_H = 0, n_h = 1$). For Class B the mass matrix can again be read from the one written

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below. We have

$$\begin{aligned} \mathcal{M}^2_{\gamma\gamma'} &= M^2_{\gamma\gamma'} + \Delta_{\gamma\gamma'}\,, \qquad \gamma,\gamma' = a,b,c,d,e\,, \\ M^2_{e\gamma} &= M^2_{\gamma e} = 0\,, \qquad \gamma = a,b,c,d,e \end{aligned} \tag{5.3}$$

with $M^2_{\alpha\beta}$ $(\alpha,\beta = a,\dots,d)$ as in (2.12) and with

$$\begin{aligned} \Delta_{bb} &= g_b^2\left[q^2_{b_{\sigma_1}}v_1^2 + q^2_{b_{\sigma_2}}v_2^2\right] = g_b^2\left(v_1^2+v_2^2\right), \qquad \langle\phi\rangle^2 = v_1^2+v_2^2. \\ \Delta_{bc} &= \Delta_{cb} = g_b g_c\left[q_{b_{\sigma_1}}q_{c_{\sigma_1}}v_1^2 + q_{b_{\sigma_2}}q_{c_{\sigma_2}}v_2^2\right] = \delta\, g_b g_c\left(v_1^2+v_2^2\right), \\ \Delta_{cc} &= g_c^2\left[q^2_{c_{\sigma_1}}v_1^2 + q^2_{c_{\sigma_2}}v_2^2\right] = g_c^2\left(v_1^2+v_2^2\right), \\ \Delta_{be} &= \Delta_{eb} = g_b g_L\left[T_3(\sigma_1)q_{b_{\sigma_1}}v_1^2 + T_3(\sigma_2)q_{b_{\sigma_2}}v_2^2\right] = -\delta\, g_b g_L/2\left(v_1^2+v_2^2\right), \\ \Delta_{ce} &= \Delta_{ec} = g_c g_L\left[T_3(\sigma_1)q_{c_{\sigma_1}}v_1^2 + T_3(\sigma_2)q_{c_{\sigma_2}}v_2^2\right] = -g_c g_L/2\left(v_1^2+v_2^2\right), \\ \Delta_{ee} &= g_L^2\left[T_3^2(\sigma_1)\,v_1^2 + T_3^2(\sigma_2)v_2^2\right] = g_L^2/4\left(v_1^2+v_2^2\right). \end{aligned} \tag{5.4}$$

with $\delta = +1$ if $\sigma_i = H_i$ (Class A, $n_H = 1, n_h = 0$) and $\delta = -1$ if $\sigma_i = h_i$ (Class A, $n_H = 0, n_h = 1$), respectively. Class B of D6 models has a Higgs sector which is a superposition of both possibilities above, with appropriate choice for the v.e.v. of the Higgs fields involved. For Class B one formally replaces $\delta \to \cos(2\theta)$ with θ the mixing between h_i and H_i sectors, to be detailed later (formally $\theta = 0, \pi/2$ for Class A). For the D5-brane model discussed before, the electroweak sector is that of class A models with $\delta = +1$. All matrices $(\mathcal{M}^2, \Delta)$ are symmetric, with remaining entries of Δ not defined above, equal to zero. The mass eigenvalue equation for $\mathcal{M}^2$ gives (see also (3.10))

$$\lambda\left(\lambda^4 + o_3\lambda^3 + o_2\lambda^2 + o_1\lambda + o_0\right) = 0\,. \tag{5.5}$$

Thus the matrix $\mathcal{M}^2_{\gamma\gamma'}$ has a zero mass eigenvalue corresponding to the photon eigenstate. Its eigenvector is a simple extension of that of hypercharge eigenvector w_1 given in eqs. (4.3) and (4.13):

$$e_1 = \frac{1}{|e_1|}\left\{\frac{g_d}{3g_a}, 0, -\frac{g_d}{g_c}, 1, -\frac{g_d}{g_b}\right\}_\alpha\,. \tag{5.6}$$

This confirms that the anomalous $\mathrm{U}(1)_b$ field is not part of the photon eigenstate, as expected.

We now compute the root of eq. (5.5) corresponding to M_Z. The remaining roots which are electroweak corrections to (stringy) masses of U(1) fields of previous sections can be computed in exactly the same way. Although the method we use to computing M_Z is similar to D6 (Class A and B) and D5-brane models as well, we consider these cases separately. This is done because the Higgs sector is different and because it is our intention to compare separately the results of D6, D5 models with their corresponding cases before EWS breaking.

5.1 D6-brane models. stringy corrections to M_Z and bounds on M_S

5.1.1 Class A models.

For Class A models of table 6 one may compute the roots of eq. (5.5) explicitly. Analytic formulae for these exist, but they are long and not very enlightening. Instead one can compute these solutions as expansions in η about their values in the absence of the electroweak symmetry breaking. We take account that the coefficients o_0, o_1, o_2, o_3 of (5.5) contain corrections linear in η in addition to their values in the absence of the EWS breaking (represented by the coefficients in eqs. (4.2)). In Class A models the coefficients o_i are

$$\begin{aligned} o_0 &= \eta\, s_0\,, \qquad \eta \equiv \langle\phi\rangle^2/M_S^2\,, \\ o_i &= c_i + \eta\, s_i\,, \qquad i = 1,2,3\,. \end{aligned} \tag{5.7}$$

s_0, s_i are independent on $\langle\phi\rangle$ ($o_i \to c_i$ if $\langle\phi\rangle \to 0$).

$$\begin{aligned} s_0 &= \frac{4\epsilon^4 g_b^2\, n_{c1}^2}{\beta_2^2\nu^2}\left\{g_b^2 g_c^2 g_d^2 + 9\, g_a^2\left[g_c^2 g_d^2 + g_b^2\left(g_c^2+g_d^2\right)\right]\right\}, \\ s_1 &= \frac{\epsilon^2}{\beta_1^2\beta_2^4\nu^2}\Big\{\beta_1^2 g_b^2\nu^2\left[4\beta_1\beta_2\left(g_b^2+g_c^2\right)g_d^2 n_{a2}n_{c1} - 4\beta_1^2\left(g_c^2 g_d^2 + g_b^2\left(g_c^2+g_d^2\right)\right)n_{c1}^2\right] + \\ &\quad + \beta_2^4\left[6 g_b^2 g_c^2 (9g_a^2+g_d^2) n_{b1}n_{c1}\nu\delta - n_{c1}^2\left(2g_b^2 g_c^2 g_d^2 + 9g_a^2\left(g_c^2 g_d^2 + 2g_b^2(g_c^2+g_d^2)\right)\right)\right] - \\ &\quad - \beta_2^2\, g_b^2\left(g_b^2+g_c^2\right)\left(9g_a^2+g_d^2\right)\left(4\,\beta_1^2\beta_2^2\epsilon^2 + n_{a2}^2\nu^2\beta_1^2 + 9\beta_2^2\nu^2 n_{b1}^2\right)\Big\}, \\ s_2 &= \frac{1}{4\beta_1^2\beta_2^2\nu^2}\left[\left(9g_a^2+g_d^2\right)\left(4\beta_2^2\epsilon^2 + n_{a2}^2\nu^2\right)\beta_2^2 - 4\beta_1\beta_2 g_d^2 n_{a2}n_{c1}\nu^2\right]\left(2g_b^2+g_c^2\right) + \frac{g_c^2 g_d^2 n_{c1}^2}{\beta_2^2} + \\ &\quad + \frac{g_b^2}{\beta_2^2}\left[2\left(g_c^2+g_d^2\right)n_{c1}^2 - 6g_c^2 n_{b1}n_{c1}\nu\delta + \left(4\beta_1^2\epsilon^2 + 9n_{b1}^2\nu^2\right)\left(g_b^2+g_c^2\right)\right], \\ s_3 &= -2g_b^2 - g_c^2\,. \end{aligned} \tag{5.8}$$

One eigenvalue in (5.5) is $\lambda_0 = 0$ with the remaining ones denoted by λ_i with $i = 2,3,4,5$. In the limit $\eta \to 0$, $\lambda_{2,3,4}$ are equal to those in the absence of the electroweak symmetry breaking (and non-zero) see eqs. (3.5) and (4.2). Their electroweak corrections are suppressed by powers of η relative to their non-vanishing value in the absence of EWS breaking, are very small and we will not address them further. For our purpose, we are interested in the remaining λ_5 which is the *only* one to vanish *in the limit* $\eta \to 0$, and may thus be approximated as an expansion in positive powers of η (no constant term). This is just the mass of the Z boson. We thus search for a solution to M_Z of the type

$$M_Z^2 \equiv \lambda_5 M_S^2 = \left[\eta\xi_1 + \eta^2\,\xi_2 + \eta^3\xi_3 + \eta^4\xi_4 + \cdots\right] M_S^2 \tag{5.9}$$

$$= \left[1 + \eta\xi_{21} + \eta^2\xi_{31} + \eta^3\xi_{41} + \cdots\right]\xi_1\langle\phi\rangle^2 \tag{5.10}$$

To compute $\xi_1, \xi_{21}, \xi_{31}$ we use eqs. (5.5), (5.7) and (5.9) together with c_i of (4.2). Discarding terms of order $\mathcal{O}(\eta^5)$ in (5.5) we find[7]

$$\begin{aligned} \xi_1 &= \left\{g_b^2 g_c^2 g_d^2 + 9g_a^2\left[g_c^2 g_d^2 + g_b^2(g_c^2+g_d^2)\right]\right\}\left[g_c^2 g_d^2 + 9g_a^2(g_c^2+g_d^2)\right]^{-1} \\ &= \frac{1}{4}\left(4g_b^2 + g_y^2\right), \end{aligned} \tag{5.11}$$

[7]The definition of ξ_1, ξ_{21}, ξ_{31} in terms of c_i and s_i was outlined in (3.14).

where in the last step we used the hypercharge coupling definition eq. (4.8). For ξ_{21} we find

$$\begin{aligned}\xi_{21} = &-\Big\{4\beta_2^4\beta_1^2\epsilon^2 g_c^4(9g_a^2+g_d^2)^2+\beta_2^4\left[\left(g_c^2g_d^2+9g_a^2\left(g_c^2+g_d^2\right)\right)n_{c1}-3g_c^2 n_{b1}\nu\delta\left(9g_a^2+g_d^2\right)\right]^2+\\ &+\beta_1^2 g_c^4\nu^2\left[\beta_2 n_{a2}(9g_a^2+g_d^2)-2\beta_1 g_d^2 n_{c1}\right]^2\Big\}\times\\ &\times\Big\{4\beta_1^2\beta_2^2\epsilon^2\left[g_c^2g_d^2+9g_a^2(g_c^2+g_d^2)\right]^2 n_{c1}^2\Big\}^{-1}.\end{aligned} \tag{5.12}$$

Since $\xi_{21}<0$ the mass M_Z (5.10) will be smaller than that of the SM Z boson. Note that ξ_{21} is similar for all models with same n_{a2}, β_2 of Class A defined in table 6 (except the term in the second square bracket depending on n_{c1}). The first (squared) square bracket in ξ_{21} is similar for all models of Class A, since $n_{c1}n_{b1}\nu=-1$ is invariant for all models. This will imply similar behaviour for all models of Class A that we address.

In terms of the chosen parameters of Class A models, the ratio $\mathcal{R}=g_d/g_c$ and n_{a2}, ξ_{21} may be re-written as

$$\begin{aligned}\xi_{21} = &-\Big\{\beta_1^2\left[2\beta_1 g_y^2\nu n_{c1}(1+\mathcal{R}^2)-(36g_a^2+g_y^2\mathcal{R}^2)\beta_2 n_{a2}\nu\right]^2+4\beta_1^2\beta_2^4\epsilon^2\left(36g_a^2+g_y^2\mathcal{R}^2\right)^2+\\ &+9\beta_2^4\left[g_y^2 n_{b1}\nu\mathcal{R}^2+12g_a^2\left[3n_{b1}\nu-n_{c1}(1+\mathcal{R}^2)\delta\right]\right]^2\Big\}\times\\ &\times\left[5184\beta_1^2\beta_2^2\epsilon^2 g_a^4 n_{c1}^2\left(1+\mathcal{R}^2\right)^2\right]^{-1}\end{aligned} \tag{5.13}$$

and this will be used in the last part of this section. Finally, the expression of ξ_{31} is

$$\begin{aligned}\xi_{31} = &\left(16\beta_1^4\beta_2^4\epsilon^4 g_b^4 a_3^4 n_{c1}^4\right)^{-1}\times\\ &\times\left[2a_0^2a_1^2+3a_0a_1a_2a_3-\beta_1^2\beta_2^4 g_b^2 a_0^2 a_3 a_4+a_3^2(\beta_1^2\beta_2^4 g_b^2 a_0 a_5+a_2^2)\right],\end{aligned} \tag{5.14}$$

where the coefficients a_i (i=1,...5) are given in appendix B. Unlike ξ_{21}, ξ_{31} has no definite sign.[8] Numerical investigations for all models of this class show that the effects of ξ_{31} are very small relative to those induced by ξ_{21} and this ensures our procedure is rapidly convergent.

We therefore find that

$$\begin{aligned}M_Z^2 &= \frac{1}{4}\left(4g_b^2+g_y^2\right)\langle\phi\rangle^2\left[1+\eta\xi_{21}+\eta^2\xi_{31}+\cdots\right]\\ &= M_{Z,0}^2\left[1+\eta\xi_{21}+\eta^2\,\xi_{31}+\cdots\right]\end{aligned} \tag{5.15}$$

which in the lowest order in η recovers the usual mass formula of Z boson ($4g_b^2=g_L^2$) induced by the Higgs mechanism alone. ξ_{21} does not depend on the coupling $g_b(g_L)$ as this (electroweak) dependence is factorised into the first term in the expansion of M_Z. Higher corrections (ξ_{31}) depend on g_b since Higgs state was charged under $U(1)_b$, $U(1)_c$

[8] For a Class A model with $\beta_1=1/2$, $\nu=1/3$, $\beta_2=1$, $n_{c1}=1$, $n_{b1}=-1$, $\delta=+1$ numerical investigations show that if $g_d/g_c\approx 0.7$ or less, and with n_{a2} taking values smaller than 20, $\xi_{31}>0$ and has an opposite effect to ξ_{21}, to increase M_Z. For $g_d/g_c>0.7$, $\xi_{31}<0$, and its effects add to those of ξ_{21} to decrease M_Z. Generically $\xi_{21}=\mathcal{O}(1)$ up to $\mathcal{O}(10)$ and the ratio ξ_{31}/ξ_{21} is of order $\mathcal{O}(1)$, except cases with $g_d/g_c<1$ for large n_{a2} when the ratio may become larger, up to $\mathcal{O}(10)$.

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(which also mix) see table 5. The difference between models of Class A with $n_H = 1, n_h = 0$ or with $n_H = 0, n_h = 1$ is marked in eqs. (5.12) and (5.14) defining ξ_{21} and ξ_{31} by the presence of $\delta = +1$ and $\delta = -1$, respectively.

The above correction to M_Z is due to the initial presence of additional $\mathrm{U}(1)_\alpha$'s and their mass in the absence of EWS breaking, eq. (5.1) induced by the couplings of their field strength tensor to the RR two form fields, $F \wedge B$. Thus the correction (5.15) is *not* due to the Higgs mechanism. Given the string origin of the couplings $F \wedge B$, the additional correction to M_Z, as a result of mixing with the $\mathrm{U}(1)_\alpha$ bosons, is itself of string nature and there is no Higgs particle associated with it. This mechanism is interesting in itself, regardless of the phenomenological viability of the models.

From equation (5.15) we can now extract (lower) bounds on the value of the string scale for all possible models of Class A, table 6. This is done by using the definition of the ρ parameter

$$\rho = \frac{M_W^2}{M_Z^2 \cos\theta_W} \tag{5.16}$$

with $\rho = \rho_0$ for SM case. Experimental constraints give that for a Higgs mass[9] at 115 GeV [17]

$$\frac{\Delta\rho}{\rho_0} = \frac{\pm 0.0006}{1.0004} \tag{5.17}$$

and this will be used in the following to impose bounds on M_S. From eq. (5.15) for corrected Z mass we find that

$$\frac{\Delta\rho}{\rho_0} = -1 + \frac{1}{1 + \eta\xi_{21} + \eta^2\,\xi_{31} + \mathcal{O}(\eta^3)} \tag{5.18}$$

which may be solved for η in terms of $\Delta\rho/\rho_0$. From (5.18), the (lower) bound on M_S is given by

$$M_S^2 = \langle\phi\rangle^2(-\xi_{21})\left[1 + \frac{\rho^0}{\Delta\rho}\right]\left[1 - 2\frac{\xi_{31}}{\xi_{21}^2}\frac{\Delta\rho/\rho^0}{1 + \Delta\rho/\rho^0}\right]^{1/2}. \tag{5.19}$$

Since the correction to M_S due to ξ_{31} (relative to the case when it is ignored) is very small (less than 0.1%) we can safely neglect it (also $\rho_0/\Delta\rho \gg 1$) and keeping the lowest order in η one finds

$$M_S^2 \approx \langle\phi\rangle^2(-\xi_{21})\frac{\rho_0}{\Delta\rho}\,. \tag{5.20}$$

Since $\xi_{21} < 0$, M_Z is decreased from its SM value, so $\Delta\rho = \rho - \rho_0 > 0$. Therefore the positive correction in (5.17) is considered. Equation (5.20) shows that the (lower) bound on M_S is raised as the uncertainty $\Delta\rho$ of measuring ρ_0 is reduced. With our choice for the ρ parameter consistent with $M_H = 115\,\mathrm{GeV}$ eq. (5.20) gives

$$M_S \approx 10046.7 \times |\xi_{21}|^{1/2}\,\mathrm{GeV}. \tag{5.21}$$

Using expression (5.19) or (5.18) one can derive bounds on M_S in terms of the parameters

[9] We use the value of the ρ parameter which complies with this value for the Higgs mass. One can ignore this constraint, and consider the value of $\rho = 1.0012 + 0.0023/-0.0014$ [17] whose implications (of decreasing M_S by ≈ 1.96) for our findings may easily be recovered from eq. (5.20).

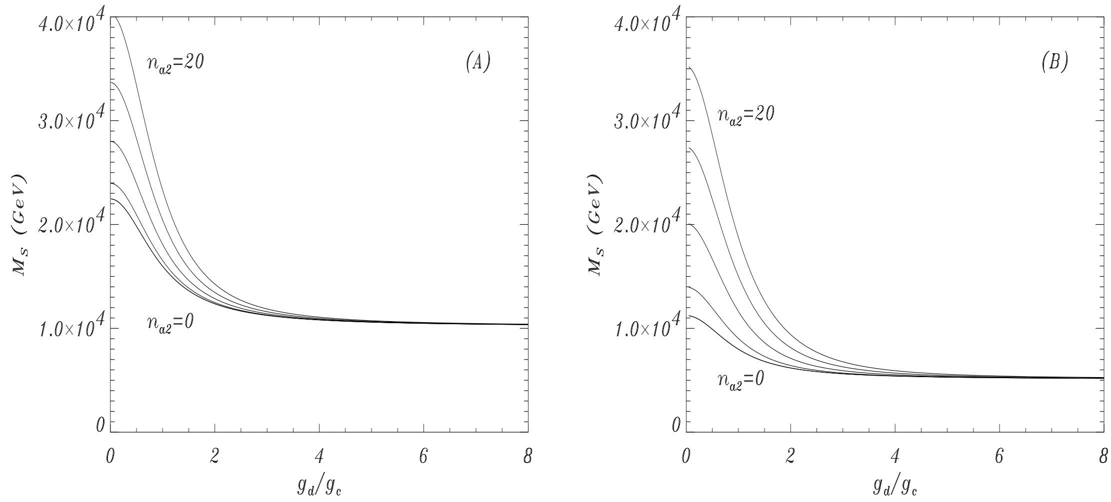

Figure 6: D6-brane models: Class A models. (A): Lower bounds on the string scale, M_S (GeV), in function of the ratio g_d/g_c, with n_{a2} fixed for each curve. n_{a2} is increasing upwards (step 5) from $n_{a2} = 0$ (lowest curve) to $n_{a2} = 20$. The plots correspond to a model of Class A defined by $\beta_1 = 1/2$, $\nu = 1/3$, $\beta_2 = 1$, $n_{c1} = 1$, $n_{b1} = -1$, $\delta = +1$. Lowest M_S is 10 TeV. (B): As for (A) but $\beta_2 = 1/2$ (also $\nu = 1/3$, $\beta_1 = 1/2$, $n_{c1} = 1$, $n_{b1} = -1$, $\delta = +1$). The lowest allowed M_S can be as small as 5 TeV. This value and that in (A) may further be decreased, by a factor of up to 1.96 when relaxing the constraints on M_H, see text.

of the model (ratio g_d/g_c and n_{a2}). These are in very good agreement ($< 2\%$ for M_S) with the one derived using a full numerical approach to finding the value of M_Z from $\mathcal{M}^2_{\alpha\beta}$ of (5.5) and from that constraints on M_S.

We start with the first example of Class A models of table 6. For this particular model with: $\delta = +1$, $\nu = 1/3$, $\beta_1 = 1/2$, $\beta_2 = 1$, $n_{b1} = -1$, $n_{c1} = 1$, figure 6A presents the numerical dependence of the string scale in function of g_d/g_c for various n_{a2}. The plot takes into account that g_d and g_c are not independent, but correlated to g_y (via eq. (4.8)) which is kept fixed. The lower bounds on M_S are in the region of 10 TeV for large (parameter) ratio g_d/g_c. These bounds increase significantly up to a range of 22–40 TeV, as n_{a2} is increased from 0 to 20, for regions with the ratio of couplings $g_d/g_c < 1$. All models of Class A (table 6) which differ from this example only by n_{c1}, n_{b1}, δ, with $\beta_1 = 1/2$, $\beta_2 = 1$, $\nu = 1/3$, have very similar dependence with respect to g_d/g_c (for given n_{a2}) and the same bounds on M_S, as in figures 6A and 6B. This similarity may be noticed from a careful analysis of ξ_{21} which sets the behaviour of M_S.

We note the large spreading with respect to n_{a2} for fixed $g_d/g_c < 1$ (thus the wide range of values for M_S) in figure 6. For similar ratios of g_d/g_c and range of n_{a2} values as in this figure, the amount of $\mathrm{U}(1)_b$ in the mass eigenstates M_4 has a significant effect (spread) with respect to n_{a2} (similar case for M_2 but less significant), while in M_3 this effect is small. Note that very approximately M_S is proportional to the product of M_2 and M_4. The spreading/sensitivity of M_S with respect to n_{a2} for fixed g_d/g_c in figure 6 is then induced via M_4. As we increase the ratio g_d/g_c, the spreading with respect to n_{a2} disappears, and $\mathrm{U}(1)_b$ dominates in M_3 but not in $M_{2,4}$.

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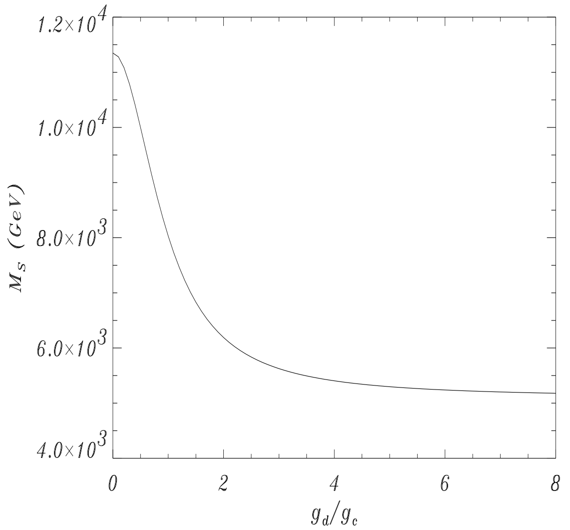

Figure 7: D6-brane models: The string scale M_S (GeV), in function of the ratio g_d/g_c, corresponding to a model of Class A, line 3 of the table 6, parametrised by $\nu = 1/3$, $\beta_1 = 1/2$, $\beta_2 = 1/2$, $n_{a2} = n_{b1} = n_{c1} = n_{d2} = 1$.

As another example we show in figure 7 the constraints for a model corresponding to the third line of table 6, defined by $\nu = 1/3$, $\beta_1 = 1/2$, $\beta_2 = 1/2$, $n_{a2} = n_{b1} = n_{c1} = n_{d2} = 1$. As mentioned in section 2.3 such a model may be "approximately" supersymmetric in the sense discussed in [13]. With all these parameters fixed, only g_d/g_c remains a free parameter and we can plot the limit on the value of the string scale M_S as a function of g_d/g_c. In this case we must have $M_S \geq 5\,\mathrm{TeV}$ in order to be within the experimental limits for ρ.

As a general conclusion one finds that constraints from the ρ-parameter are somewhat stronger than those obtained from direct searches. Very often the constraints imply the need of rather high (of order 10–20 TeV) values for the string scale. Still, we find that the string scale can have values as low as 5 TeV, figure 6B, while still accommodating the constraints from the ρ parameter. These values of the string scale may further decrease by a factor of ≈ 1.96 if the uncertainty in the ρ parameter that we used $(+0.0006)$ throughout this analysis and in figures 6 and 7 was relaxed to $(+0.0023)$ [17] which does not take account of the constraint $M_H = 115\,\mathrm{GeV}$. This would lead to M_S as low as 2.5 TeV for (some) Class A models.

5.1.2 Class B models.

For class B models of table 6 we use an approach similar to Class A models to find lower bounds on the value of the string scale. However, eq. (5.7) has a different structure, as corrections $\mathcal{O}(\eta^2)$ to c_i are now present in the definition of o_i and care must be taken in finding the analytical expansion in η for M_Z. For this one may use the expansion outlined in eq. (3.14).

In Class B models there are two Higgs sectors, H_i (i=1,2) with vev's $v_i/\sqrt{2}$ and h_i $(i=1,2)$ with vev's $\overline{v}_i/\sqrt{2}$. We therefore need to introduce a further parameter (in addition to g_d/g_c and n_{a2}), the angle θ defined as the mixing between the two Higgs sectors (H_i and h_i, respectively):

$$\begin{aligned} \langle\phi\rangle^2 &\equiv v_1^2+v_2^2=\langle\Phi\rangle^2\cos^2\theta\,, \\ \langle\overline{\phi}\rangle^2 &\equiv \overline{v}_1^2+\overline{v}_2^2=\langle\Phi\rangle^2\sin^2\theta\,. \end{aligned} \tag{5.22}$$

As in Class A one can compute the corrections to M_Z as an expansion in the parameter $\langle\Phi\rangle^2/M_S^2$. Using eqs. (5.3) , (5.4) and (5.22) one finds

$$\begin{aligned} M_Z^2 &= \left[1+\eta'\xi_{21}+\eta'^2\xi_{31}+\cdots\right]\xi_1\langle\Phi\rangle^2\,, \qquad \eta'=\langle\Phi\rangle^2/M_S^2 \\ &= \frac{1}{4}\left(4g_b^2+g_y^2\right)\langle\Phi\rangle^2\left[1+\eta'\xi_{21}+\eta'^2\xi_{31}+\cdots\right] \\ &\equiv M_{Z,0}^2\left[1+\eta'\,\xi_{21}+\eta'^2\xi_{31}+\cdots\right]. \end{aligned} \tag{5.23}$$

In (5.23) we used eq. (4.16) to re-write ξ_1 in terms of the hypercharge coupling:

$$\begin{aligned} \xi_1 &= \left\{g_b^2g_c^2g_d^2+9g_a^2\left[g_c^2g_d^2+g_b^2\left(g_c^2+g_d^2\right)\right]\right\}\times \\ &\quad\times\left[g_c^2g_d^2+9g_a^2\left(g_c^2+g_d^2\right)\right]^{-1}=\frac{1}{4}\left(4g_b^2+g_y^2\right). \end{aligned} \tag{5.24}$$

We also have that

$$\begin{aligned} \xi_{21} &= -\Big\{4\beta_2^4\beta_1^2\epsilon^2g_c^4\left(9g_a^2+g_d^2\right)^2+\beta_2^4n_{c1}^2\left(g_c^2g_d^2+9g_a^2(g_c^2+g_d^2)\right)^2\cos^2(2\theta)+ \\ &\quad+\beta_1^2g_c^4\nu^2\left[\beta_2(9g_a^2+g_d^2)n_{a2}-2\beta_1g_d^2n_{c1}\right]^2\Big\}\times \\ &\quad\times\Big\{4\beta_1^2\beta_2^2\epsilon^2\left[g_c^2g_d^2+9g_a^2(g_c^2+g_d^2)\right]^2n_{c1}^2\Big\}^{-1}<0 \end{aligned} \tag{5.25}$$

which has an expression very similar to that of Class A, see eq. (5.12). As in Class A, $\xi_{21}<0$ thus M_Z is decreased from its SM expression. ξ_{21} may be re-written in terms of the (chosen) parameters ($\mathcal{R}=g_d/g_c$, n_{a2}, θ) as

$$\begin{aligned} \xi_{21} &= -\Big\{\beta_1^2\left[2\beta_1g_y^2\nu n_{c1}(1+\mathcal{R}^2)-(36g_a^2+g_y^2\mathcal{R}^2)\beta_2n_{a2}\nu\right]^2+4\beta_1^2\beta_2^4\epsilon^2(36g_a^2+g_y^2\mathcal{R}^2)^2+ \\ &\quad+1296\,\beta_2^4g_a^4n_{c1}^2(1+\mathcal{R}^2)^2\cos^2(2\theta)\Big\}\left[5184\,\beta_1^2\beta_2^2\epsilon^2g_a^4n_{c1}^2\left(1+\mathcal{R}^2\right)^2\right]^{-1} \end{aligned} \tag{5.26}$$

which will be used in the following. Finally, for the coefficient ξ_{31} we have

$$\xi_{31}=g_c^2\left[b_4+b_7-b_5+b_6\cos(4\theta)\right]\left(32\beta_1^4\beta_2^4\epsilon^4g_b^2n_{c1}^4b_1^4\right)^{-1}, \tag{5.27}$$

where the coefficients b_i $(i=1,\ldots 7)$ are presented in appendix B. Unlike ξ_{21} which is of definite sign, ξ_{31} is either positive or negative in function of the ratio g_d/g_c and the value of n_{a2}. Considerations similar to those made in Class A for the expansion in η' apply here as well. The accuracy of this analytical approach compared to the full numerical one to computing (the corrections to) M_Z from (5.5)) and from these the bounds on M_S, is within 2% for the string scale prediction.

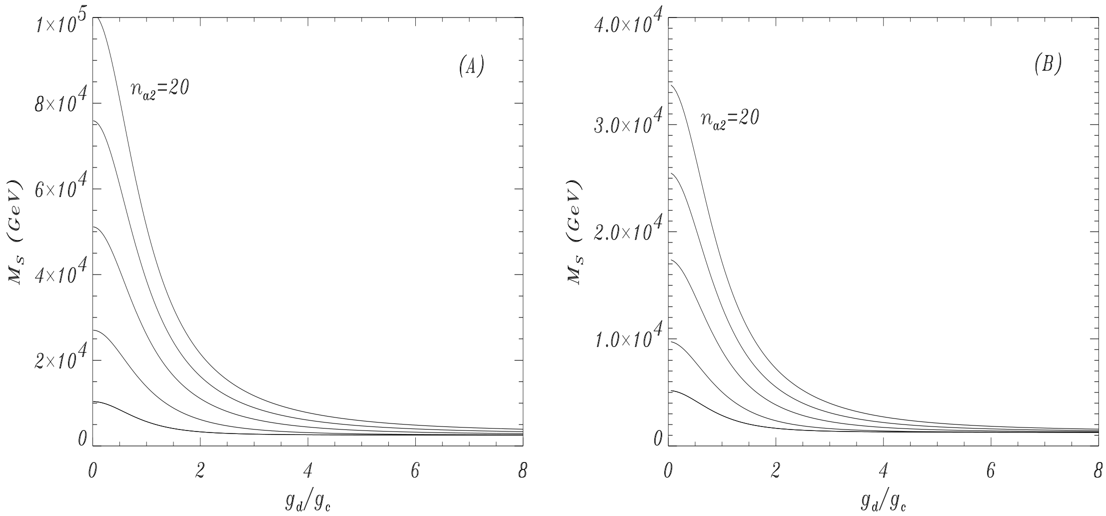

Figure 8: D6 brane models. Class B models: (A): The value of the string scale, M_S (GeV), in function of the ratio g_d/g_c, with n_{a2} fixed for each curve and increasing upwards (step 5) from $n_{a2} = 0$ (lowest curve) to $n_{a2} = 20$. The lowest M_S for which correction to M_Z is within the experimental error for the ρ parameter is ≈ 2.5 TeV. The model is defined by $\nu = 1$, $\beta_1 = 1$, $\beta_2 = 1$, $n_{c1} = 1$, $n_{b1} = 0$, $\theta = \pi/6$. (B): As for (A) but with $\nu = 1/3$ (also $\beta_1 = 1$, $\beta_2 = 1/2$, $n_{c1} = 1$, $n_{b1} = 0$, $\theta = \pi/6$). The change in ν brings a range for M_S similar to that of Class A models, where this parameter has the same value. Lowest M_S which still respects ρ constraints is ≈ 1.5 TeV.

Using the same procedure as in Class A, we find a similar expression for the string scale bound

$$M_S^2 = \langle\Phi\rangle^2(-\xi_{21})\left[1+\frac{\rho_0}{\Delta\rho}\right]\left[1-2\frac{\xi_{31}}{\xi_{21}^2}\frac{\Delta\rho/\rho_0}{1+\Delta\rho/\rho_0}\right]^{1/2} \approx \langle\Phi\rangle^2(-\xi_{21})\frac{\rho_0}{\Delta\rho} \qquad (5.28)$$

and where ξ_{21} also depends on the angle θ, the direction of the vev solution.

The constraints on the string scale obtained in this way for two examples with $\nu = 1, 1/3$ see table 6, are shown in figures 8A and 8B. One may check that these figures are very similar for all other models of Class B. They show the range of bounds on M_S, which in some cases may be slightly different from Class A models. This difference is mainly due to the value of ν parameter. In figure 8B, as in Class A models, $\nu = 1/3$ and somewhat similar bounds on M_S apply. However, in figure 8A $\nu = 1$ which gives higher (lower) bounds on M_S than in Class A models. This is consequence of (5.28) and definition of ξ_{21}. Figure 8 also shows that if coupling g_d is much smaller than g_c, the bounds on M_S are not as low as when they are equal or if $g_d \gg g_c$. In the latter case these bounds can be as low as 2.5 TeV (figure 8A) or 1.5 TeV (figure 8B) with little dependence on n_{a2}.

As a general conclusion one finds that M_S may be as small as 1.5–2.5 TeV. Again, we should mention that if we relax the constraint on $\Delta\rho$ that the Higgs state have a mass equal to 115 GeV, the above lower bounds on M_S will decrease (as discussed for Class A models) by a factor of ≈ 1.96, to give bounds on M_S as low as ≈ 1 TeV for figures 8A and 8B. In such case constraints from the value of the masses $M_{2,3,4}$ be larger than $\approx$ 500–800 GeV may become stronger (but also n_{a2} dependent) because M_S can be as large as 5 times M_4 (for $g_d \gg g_c$) see figure $4F$ ($n_{a2} = 10$). This would give a value for M_S larger than 2.5–4 TeV.

5.2 D5-brane models. Stringy Corrections to M_Z and bounds on M_S

The D5-brane model that we address here is that referred to in section 4.2 in the absence of the electroweak effects. For this model we proceed as in D6-brane models and search for a solution to M_Z as an expansion in $\eta = \langle\phi\rangle^2/M_S^2$. The electroweak correction to the matrix $M^2_{\alpha\beta}$ examined in this model is identical to that of Class A of D6 models with $\delta = +1$ (see eq. (5.4)) since the Higgs sector is similar and contains only H_i.

The mass eigenvalue equation of (5.5) has new coefficients o_i with electroweak corrections to their corresponding value c_i of (4.19). These corrections contain only terms linear in η. Using an approach similar to that of previous sections we find that

$$M_Z^2 = \lambda_5 M_S^2 = M_{Z,0}^2 \left[1 + \eta\, \xi_{21} + \eta^2\, \xi_{31} + \cdots\right], \tag{5.29}$$

where $M_{Z,0}$ is the usual SM Z-boson mass and ξ_{21} is given by

$$\begin{aligned}\xi_{21} = &- \left[52 g_c^4 g_d^4 + 18 g_a^2 g_c^2 g_d^2 \left(50 g_c^2 + 6 g_d^2\right) + 81 g_a^4 \left(49 g_c^4 + 12 g_c^2 g_d^2 + 3 g_d^4\right)\right] \times \\ &\times \left[18\sqrt{3}\left(g_c^2 g_d^2 + 9 g_a^2 \left(g_c^2 + g_d^2\right)\right)^2\right]^{-1} < 0\,.\end{aligned} \tag{5.30}$$

Thus M_Z is again reduced from its Standard Model value. Since the couplings g_α ($\alpha = a, \ldots, d$) are not independent, but correlated to the hypercharge coupling, we also present the expression of ξ_{21} in terms of g_y and[10] of the only independent parameter $\mathcal{R} = g_d/g_c$ of the model. All these are evaluated at the string scale. With (4.20) we have

$$\begin{aligned}\xi_{21} = &-\Big[72\, g_a^2\, g_y^2 \left(1 + 44\mathcal{R}^2 + 3\mathcal{R}^4\right) + g_y^4 \left(1 + 4\mathcal{R}^2 + 43\mathcal{R}^4\right) + \\ &+ 1296\, g_a^4 \left(49 + 3\mathcal{R}^2\left(4 + \mathcal{R}^2\right)\right)\Big]\left[23328\,\sqrt{3} g_a^4 \left(1 + \mathcal{R}^2\right)^2\right]^{-1}.\end{aligned} \tag{5.31}$$

Similarly to Class A and Class B models, lower bounds on the string scale in terms of the ratio $\mathcal{R} = g_d/g_c$ may be found from the experimental constraints on ρ parameter. The relation between the string scale, ρ and ξ_{21}, ξ_{31} found in previous cases, see eq. (5.19) holds in this case as well. The corrections due to ξ_{31} relative to the leading contribution in η due to ξ_{21} is less than 0.1% for the string scale prediction.

In figure 9 the dependence of M_S on the ratio g_d/g_c shows the lower bounds on the string scale which still comply with the experimental constraints on the parameter ρ. The bounds on M_S can be as low as 3 TeV for large ratio g_d/g_c. This (lower) bound derived from constraints on ρ is stronger than those derived from using the (bounds on the) mass of the lightest gauge boson M_3 (section 4.2) which gave M_S larger than 1.5–2.4 TeV. We conclude that the lower bounds on M_S are similar to those of D6-brane models. We end this chapter with a reminder of our last remark of chapter 2. We have obtained in chapters 4 and 5 bounds on the masses of the extra Z' bosons in the specific intersecting D6 and

[10] g_y is fixed from low energy physics.

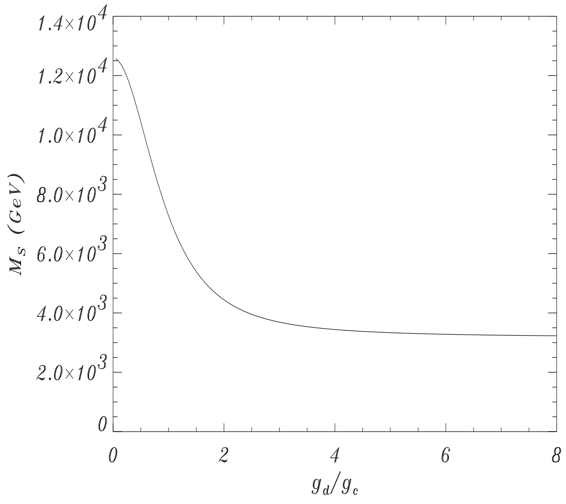

Figure 9: The string scale M_S (GeV), in function of the ratio g_d/g_c, for the D5-brane model of section 4.2. Lowest M_S is $\approx 3\,\mathrm{TeV}$ for $g_d/g_c \gg 1$.

D5-brane models reviewed in section 2. In translating these bounds into bounds on the string scale M_S we are setting possible volume factors to one. If such volume factors are very different from one, the constraints on M_S will change accordingly.

6. Conclusions

The possibility of detecting additional bosons Z' when exploring energies beyond the Standard Model scale has attracted much attention over the years, partly because it is one of the simplest extensions of the Standard Model and also because of the clean experimental signatures. String theory further increased this interest in the mid 1980's by the natural appearance of extra U(1)'s inside E_6, for instance. A thorough analysis has been performed in the past on the low energy effects of that particular class of U(1)'s.

In this paper we have studied a very well-defined class of additional massive Z bosons that appear very naturally in D-brane models. Two of them have triangle anomalies which are cancelled by a Green-Schwarz mechanism. The third additional (Abelian) gauge boson is familiar from left-right symmetric extensions of the SM. We may say that the existence of these extra U(1)'s is a generic prediction of D-brane models. If the string scale is low, these Z's should be detected very soon. Furthermore, as we have discussed in the text, these correspond to very familiar and natural U(1)'s that have not only a compelling motivation from D-brane models, but also play an important role in explaining for instance the stability of the proton.

We would like to emphasize that string theory provides a mechanism in which these gauge bosons can acquire a mass independent of the Higgs mechanism. Their mass can be understood in terms of two dual Stuckelberg mechanisms in which either the massless gauge boson "eats" a scalar or the stringy antisymmetric tensor "eats" the massless gauge boson. The important point is that for this to happen it is not necessary for a scalar field to acquire a vev and therefore the origin of their mass is not due to a Higgs field. In addition, the mixing of the U(1)'s implies that a fraction of the mass of the Standard Model Z^0 boson may be due to this "stringy" mass term, which combined with the ordinary Higgs mechanism would provide the total mass for the Z^0 boson. Such string corrections have been investigated in this paper.

We have studied the mass matrices of these U(1) fields and obtained general constraints from precision experiments on the value of their mass. Our numerical analysis has concentrated on D6- and D5-brane intersection models, but we expect analogous results for other D-brane models. We have also set possible volume factors equal to one. We have found that generically one of the extra Z_0's tends to be more massive, with mass of approximately 10 times larger than the string scale, typically the multi-TeV region. The second and the third boson may be light, within a factor varying from 1 to 10 smaller than the string scale. In some cases these states may be even lighter, depending on the exact parameters of the model (large n_{a2}). These states mix in a non-negligible manner with the physical Z_0.

Given the generic nature of these gauged U(1)'s in D-brane models, we think that the experimental search of their production at colliders is of great interest. Further, their presence could eventually be detected in precision electroweak data, as we have discussed in this paper. It would be amusing if the first hint of string theory could come from electroweak precision measurements.

Acknowledgments

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A. The Z boson eigenvector

In this Appendix we show how to compute the Z-boson eigenvector in intersecting D-brane models. We start with a comment on the "Standard Model limit". In the Standard Model case, to restore the electroweak symmetry limit, one has to take the limit $v \to 0$ in the mass formulae

$$\begin{aligned} m_W^2 &= \frac{1}{4}(g_L)^2 v^2 \,, \\ m_Z^2 &= \frac{1}{4}(g_L^2 + g_y^2) v^2 \end{aligned} \tag{A.1}$$

and at the same time the limit $g_L \to 0$ in the formulae involving mixing:

$$\cos\theta_W = \frac{g_L}{\sqrt{g_L^2 + g_y^2}} \,, \tag{A.2}$$

$$A_\gamma = \cos\theta_W B_h - \sin\theta_W W^3 \,, \tag{A.3}$$

$$A_{Z_0} = \sin\theta_W B_h + \cos\theta_W W^3 \,. \tag{A.4}$$

In this limit the Z-boson wavefunction approaches the hypercharge wavefunction:

$$A_{Z_0} \longrightarrow B_h \,. \tag{A.5}$$

This is because in the unbroken phase SU(2) is restored and the U(1) inside the SU(2) does not mix anymore with hypercharge.

Thus, one possible limit that one can take in our formulae to follow is $v \to 0$, as long as we also take $g_L \to 0$. This takes us to the regime where electroweak symmetry is not broken and therefore hypercharge is still a good symmetry. Another interesting limit is $M_S \to \infty$. This decouples the heavy states and one should arrive in this limit to the broken phase of the Standard model.

We first derive a perturbative solution to the eigenvector of Z boson. We denote this eigenvector $\mathcal{Z}$. From our perturbative solution to the eigenvalue problem, we found that the mass matrix $\mathcal{M}$ has one zero eigenvalue corresponding to the photon with eigenvector

$$\gamma = \frac{1}{|\gamma|}\left(\frac{1}{3g_a}, 0, -\frac{1}{g_c}, \frac{1}{g_d}, -\frac{2}{g_L}\right), \tag{A.6}$$

where

$$\frac{1}{|\gamma|} = \frac{1}{2}\frac{g_y g_L}{\sqrt{g_L^2 + g_y^2}} \,. \tag{A.7}$$

Then, the photon eigenstate can be written as

$$A_\gamma = \frac{g_L}{\sqrt{g_L^2 + g_y^2}}\frac{g_y}{2}\left[\frac{1}{3g_a}A_a - \frac{1}{g_c}A_c - \frac{1}{g_d}A_d\right] - \frac{g_y}{\sqrt{g_L^2 + g_y^2}}W^3 \,, \tag{A.8}$$

with W^3 the third component of the SU(2) gauge field. We can now write this in a more familiar form, namely

$$A_\gamma = \cos\theta_W B_h - \sin\theta_W W^3 \,, \tag{A.9}$$

where we have defined the hypercharge state

$$B_h = \frac{g_y}{2}\left[\frac{1}{3g_a}A_a - \frac{1}{g_c}A_c + \frac{1}{g_d}A_d\right] \tag{A.10}$$

and the Weinberg angle

$$\cos^2\theta_W = \frac{g_L^2}{g_L^2 + g_y^2} \tag{A.11}$$

as in the Standard Model. One immediate consistency check is that (A.10) corresponds precisely to the hypercharge eigenvector

$$h = \frac{g_y}{2}\left(\frac{1}{3g_a}, 0, -\frac{1}{g_c}, \frac{1}{g_d}\right). \tag{A.12}$$

The (light) eigenvalue corresponding to Z boson is

$$e_Z = \frac{1}{4}\left(g_L^2 + g_y^2\right)v^2\left(1 + \eta\xi_{21} + \mathcal{O}(\eta^2)\right). \tag{A.13}$$

We denote its associated eigenvector by

$$\mathcal{Z} = \frac{1}{|\mathcal{Z}|}(z_1, z_2, z_3, z_4, z_5)\,. \tag{A.14}$$

The remaining three eigenvalues (large) and associated eigenvectors can be approximated by their values in the absence of electroweak effects, the latter having in this case negligible effects.

To compute the eigenvector $\mathcal{Z}$ one solves the equation

$$\mathcal{M}^2 \cdot \mathcal{Z} = e_Z \cdot \mathcal{Z}\,. \tag{A.15}$$

This is a 5×5 linear system with $\det(\mathcal{M}^2) = \det(M^2 + \Delta) = 0$, which implies that one of the equations is redundant. Removing the last of them, the system that we have to solve is

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \eta Y \cdot \begin{pmatrix} 0 \\ \frac{1}{2} g_b g_L \\ \frac{1}{2} g_c g_L \\ 0 \end{pmatrix} z_5\,, \tag{A.16}$$

where

$$Y \equiv \left[M^2_{(4)} + \Delta_{(4)} - \frac{e_Z}{M_S^2} \cdot \mathbf{1}_{(4)} \right]^{-1} \tag{A.17}$$

and $M^2_{(4)}$ is the upper four by four non-vanishing sub-block of M^2 and $\Delta_{(4)}$ is the corresponding sub block of Δ (with the units divided out). The solution to this can be written as

$$z_1 = -\frac{1}{2}\eta(Y_{12} g_b + Y_{13} g_c)\,, \tag{A.18}$$

$$z_2 = -\frac{1}{2}\eta(Y_{22} g_b + Y_{23} g_c)\,, \tag{A.19}$$

$$z_3 = -\frac{1}{2}\eta(Y_{32} g_b + Y_{33} g_c)\,, \tag{A.20}$$

$$z_4 = -\frac{1}{2}\eta(Y_{42} g_b + Y_{43} g_c)\,, \tag{A.21}$$

where we have normalized $z_5 = -1/g_L$. We find the solution

$$z_1 = \frac{1}{6g_a} \frac{g_y^2}{g_y^2 - (g_L^2 + g_y^2)(1 + \eta\xi_{21})} + \frac{(g_L^2 + g_y^2)(1 + \eta\xi_{21})}{[g_y^2 - (g_L^2 + g_y^2)(1 + \eta\xi_{21})]^2} \cdot p_1 \cdot \eta + \mathcal{O}(\eta^2)\,, \tag{A.22}$$

$$z_2 = \frac{(g_L^2 + g_y^2)(1 + \eta\xi_{21})}{[g_y^2 - (g_L^2 + g_y^2)(1 + \eta\xi_{21})]^2} \cdot p_2 \cdot \eta + \mathcal{O}(\eta^2)\,, \tag{A.23}$$

$$z_3 = -\frac{1}{2g_c} \frac{g_y^2}{g_y^2 - (g_L^2 + g_y^2)(1 + \eta\xi_{21})} + \frac{(g_L^2 + g_y^2)(1 + \eta\xi_{21})}{[g_y^2 - (g_L^2 + g_y^2)(1 + \eta\xi_{21})]^2} \frac{g_y^2}{g_c} \cdot p_3 \cdot \eta + \mathcal{O}(\eta^2)\,, \tag{A.24}$$

$$z_4 = \frac{1}{2g_d} \frac{g_y^2}{g_y^2 - (g_L^2 + g_y^2)(1 + \eta\xi_{21})} + \frac{(g_L^2 + g_y^2)(1 + \eta\xi_{21})}{[g_y^2 - (g_L^2 + g_y^2)(1 + \eta\xi_{21})]^2} \cdot p_4 \cdot \eta + \mathcal{O}(\eta^2)\,, \tag{A.25}$$

where $p_1, \cdots, p_4$ are some functions of the model parameters which we do not present explicitly. To check our result we take the two limits that we mentioned in the beginning.

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- $M_S \to \infty$.

  All terms proportional to η vanish and we are left with a simple eigenvector which however has to be properly normalized first. Once we do so, after the limit we arrive at the Z eigenstate

$$A_{Z_0} = \frac{g_y}{\sqrt{g_L^2+g_y^2}}\left[\frac{g_y}{2}\left(\frac{1}{3g_a}A_a - \frac{1}{g_c}A_c + \frac{1}{g_d}A_d\right)\right] + \frac{g_L}{\sqrt{g_L^2+g_y^2}}W^3\,, \tag{A.26}$$

  which is

$$A_{Z_0} = \sin\theta_W B + \cos\theta_W W^3\,, \tag{A.27}$$

  precisely as in (A.4). To further take the limit $v \to 0$, we can proceed as in the SM case.

- $v \to 0$.

  Again, all terms proportional to η vanish. Before we take the $g_L \to 0$ limit, we have to normalize the eigenvector. If we do so, after the limit we arrive at the eigenvector

$$h = \frac{g_y}{2}\left(\frac{1}{3g_a}, 0, -\frac{1}{g_c}, \frac{1}{g_d}, 0\right)\,, \tag{A.28}$$

  which is just the hypercharge eigenvector (A.12) found previously. Actually, since the two limits commute, this is just the eigenvector that corresponds to the $g_L \to 0$ limit of (A.26).

We can write the eigenvector in a simpler form if we expand and keep only terms to $\mathcal{O}(\eta)$:

$$\mathcal{Z} = \frac{1}{|\mathcal{Z}|}\left[\mathcal{Z}_0 + \eta\cdot\mathcal{Z}'\right]\,, \tag{A.29}$$

where the first part is the Standard Model part (see (A.26))

$$\mathcal{Z}_0 = (z_{1_0}, z_{2_0}, z_{3_0}, z_{4_0}, z_{5_0}) = \left(\frac{g_y^2}{3g_a}, 0, -\frac{g_y^2}{g_c}, \frac{g_y^2}{g_d}, 2g_L\right)\,, \tag{A.30}$$

and the perturbation is

$$\mathcal{Z}' = \left(z_1', z_2', z_3', z_4', z_5'\right) \tag{A.31}$$

with

$$z_1' = \frac{1}{g_L^2\cos^2\theta_W}\left[g_y^2\xi_{21} + p_1\right]\,, \tag{A.32}$$

$$z_2' = \frac{1}{g_L^2\cos^2\theta_W}p_2\,, \tag{A.33}$$

$$z_3' = \frac{1}{g_L^2\cos^2\theta_W}\left[g_y^2\xi_{21} + \frac{g_y^2}{g_c}p_3\right]\,, \tag{A.34}$$

$$z_4' = \frac{1}{g_L^2\cos^2\theta_W}\left[g_y^2\xi_{21} + p_4\right]\,, \tag{A.35}$$

$$z_5' = 0\,. \tag{A.36}$$

Now we determine the norm to order η. Let

$$|\mathcal{Z}| = N + \eta N' \,, \tag{A.37}$$

where

$$N = 2g_y\sqrt{g_L^2 + g_y^2} \,. \tag{A.38}$$

The requirement that to order η the norm is equal to one fixes N' to

$$N' = z_{1_0} z_1' + \cdots + z_{5_0} z_5' \,. \tag{A.39}$$

Finally we can write the Z-boson eigenvector as the normalized to one SM eigenvector, plus a perturbation due to the presence of the extra U(1)'s, to order η:

$$\mathcal{Z} = \frac{1}{N}\mathcal{Z}_0 - \eta\frac{1}{N}\left(\frac{N'}{N}\mathcal{Z}_0 - \mathcal{Z}'\right) + \mathcal{O}(\eta^2) \,. \tag{A.40}$$

This corresponds to the eigenstate:

$$A_Z = A_{Z_0} - \eta\frac{1}{N}\left[N' A_{Z_0} - \left(z_1' A_a + z_2' A_b + z_3' A_c + z_4' A_d\right)\right] + \mathcal{O}\left(\eta^2\right) . \tag{A.41}$$

It is possible to express the correction in terms of the new basis A_1, A_2, A_3, A_4 instead of the A_a, A_b, A_c, A_d basis. In fact, since we are already at $\mathcal{O}(\eta)$ we can express the old basis in terms of the new basis to zero order. We finally find:

$$\begin{aligned} A_Z \;=\; & A_{Z_0}\left(1 - \eta\frac{N'}{N}\right) + \eta\frac{1}{N}\Big(z_1'(\mathcal{F})^{-1}_{ai} A_i + z_2'(\mathcal{F})^{-1}_{bi} A_i + \\ & \qquad + \; z_3'(\mathcal{F})^{-1}_{ci} A_i + z_4'(\mathcal{F})^{-1}_{di} A_i\Big) + \mathcal{O}(\eta^2) \end{aligned} \tag{A.42}$$

with sum over i understood.

B. Explicit formulae of ξ_{31} of Class $A\&B$ of D6-brane models

For Class A models, the expression of ξ_{31} of eq. (5.14) is

$$\xi_{31} = \left(16\beta_1^4\beta_2^4\epsilon^4 g_b^4 a_3^4 n_{c1}^4\right)^{-1}\left[2a_0^2a_1^2 + 3a_0a_1a_2a_3 - \beta_1^2\beta_2^4 g_b^2 a_0^2 a_3 a_4 + a_3^2\left(\beta_1^2\beta_2^4 g_b^2 a_0 a_5 + a_2^2\right)\right] ,$$

where we used the following notation:

$$\begin{aligned} a_0 \;=\; & g_b^2 g_c^2 g_d^2 + 9g_a^2\left(g_c^2 g_d^2 + g_b^2\left(g_c^2 + g_d^2\right)\right) , \\ a_1 \;=\; & {\beta_2}^4\left[\left(9g_a^2 + g_d^2\right)\left(g_c^2 n_{c1}^2 + 9g_b^2 n_{b1}^2\nu^2\right) + 9g_a^2 g_d^2 n_{c1}^2\right] + \\ & + g_b^2\beta_1^2\left[\beta_2^2\left(9g_a^2 + g_d^2\right)\left(4\beta_2^2\epsilon^2 + n_{a2}^2\nu^2\right) - 4\beta_1\beta_2 g_d^2 n_{a2}n_{c1} + 4{\beta_1}^2\left(g_c^2 + g_d^2\right) n_{c1}^2\right]\nu^2 , \\ a_2 \;=\; & {\beta_2}^4\left[-4\,{\beta_1}^2\epsilon^2 g_b^2\left(g_b^2 + g_c^2\right)\left(9g_a^2 + g_d^2\right) - \left(2g_b^2 g_c^2 g_d^2 + 9g_a^2\left(g_c^2 g_d^2 + 2g_b^2\left(g_c^2 + g_d^2\right)\right)\right){n_{c1}}^2\right] + \\ & + 6{\beta_2}^4 g_b^2 g_c^2\left(9g_a^2 + g_d^2\right) n_{b1}n_{c1}\nu\delta - \\ & - g_b^2\Big[{\beta_2}^2\left(g_b^2 + g_c^2\right)\left(9g_a^2 + g_d^2\right)\left({\beta_1}^2{n_{a2}}^2 + 9{\beta_2}^2{n_{b1}}^2\right) - \\ & \qquad - \;4\,{\beta_1}^3\beta_2\,\left(g_b^2 + g_c^2\right)\,g_d^2\,n_{a2}n_{c1} + 4\,{\beta_1}^4\,\left(g_c^2\,g_d^2 + g_b^2\,\left(g_c^2 + g_d^2\right)\right){n_{c1}}^2\Big]\nu^2, \end{aligned}$$

$$
\begin{aligned}
a_3 &= g_c^2 g_d^2 + 9g_a^2\left(g_c^2+g_d^2\right),\\
a_4 &= n_{c1}^2\left[4\beta_2^4\epsilon^2\left(9g_a^2+g_d^2\right)+\beta_2^2 n_{a2}^2\nu^2\left(9g_a^2+g_d^2\right)-4\beta_1\beta_2 g_d^2 n_{a2}n_{c1}\nu^2+\right.\\
&\qquad\left.+\ 4\beta_1^2\nu^2\left(4\beta_1^2\epsilon^2 g_b^2+\left(g_c^2+g_d^2\right)n_{c1}^2+9g_b^2 n_{b1}^2\nu^2\right)\right],\\
a_5 &= {n_{c1}}^2\left\{4\,{\beta_2}^4\,\epsilon^2\,\left(2\,g_b^2+g_c^2\right)\,\left(9\,g_a^2+g_d^2\right)+{\beta_2}^2\,\left(2\,g_b^2+g_c^2\right)\,\left(9\,g_a^2+g_d^2\right)\,{n_{a2}}^2\,\nu^2-\right.\\
&\qquad-\ 4\,\beta_1\,\beta_2\,\left(2\,g_b^2+g_c^2\right)\,g_d^2\,n_{a2}\,n_{c1}\,\nu^2+\\
&\qquad+\ 4\,{\beta_1}^2\,\nu^2\,\left[4\,{\beta_1}^2\,\epsilon^2\,g_b^2\,\left(g_b^2+g_c^2\right)+\left(g_c^2\,g_d^2+2\,g_b^2\,\left(g_c^2+g_d^2\right)\right)\,{n_{c1}}^2-\right.\\
&\qquad\qquad\left.\left.-\ 6\,g_b^2\,g_c^2\,n_{b1}n_{c1}\nu\,\delta+9\,g_b^2\,\left(g_b^2+g_c^2\right)\,{n_{b1}}^2\nu^2\right]\right\}.
\end{aligned}
$$

For Class B we have for ξ_{31} of eq. (5.27):

$$
\xi_{31} = g_c^2\left[b_4+b_7-b_5+b_6\cos(4\theta)\right]\left(32\beta_1^4\beta_2^4\epsilon^4 g_b^2 n_{c1}^4 b_1^4\right)^{-1},
$$

where we used the following notation:

$$
\begin{aligned}
b_7 &= 2\beta_1^2\beta_2^4 g_b^2 g_c^2 b_2^2\left[-8\beta_1^2\epsilon^2(9g_a^2+g_d^2)b_0+b_1^2 n_{c1}^2\right]\nu^2,\\
b_6 &= \beta_2^4 b_1^2 n_{c1}^2\left[\beta_2^4\left(8\beta_1^2\epsilon^2 g_b^2 g_c^2(9g_a^2+g_d^2)^2-9g_a^2 g_d^2 b_1 n_{c1}^2\right)+2\beta_1^2 g_b^2 g_c^2 b_1\nu^2\right],\\
b_5 &= 2\beta_1^4 g_c^2 g_b^2 b_2\nu^4\left[\beta_2^2(9g_a^2+g_d^2)b_0 n_{a2}^2-4\beta_1\beta_2 g_d^2 b_0 n_{a2}n_{c1}+4\beta_1^2 n_{c1}^2 b_3\right],\\
b_4 &= \beta_2^8\left[8\beta_1^2\epsilon^2 g_b^2 g_c^2(9g_a^2+g_d^2)^2(-4\beta_1^2\epsilon^2(9g_a^2+g_d^2)b_0+b_1^2 n_{c1}^2)-9g_a^2 g_d^2 b_1^3 n_{c1}^4\right],\\
b_3 &= -g_c^4 g_d^4+g_b^2 g_c^2 g_d^2(g_c^2+g_d^2)+9g_a^2\left(g_c^2+g_d^2\right)\left[g_c^2 g_d^2+g_b^2\left(g_c^2+g_d^2\right)\right],\\
b_2 &= \left[\beta_2\left(9g_a^2+g_d^2\right)n_{a2}-2\beta_1 g_d^2 n_{c1}\right]^2,\\
b_1 &= g_c^2 g_d^2+9g_a^2\left(g_c^2+g_d^2\right),\\
b_0 &= g_c^2 g_d^2\left(g_b^2-g_c^2\right)+9g_a^2\left[-g_c^4+g_c^2 g_d^2+g_b^2\left(g_c^2+g_d^2\right)\right].
\end{aligned}
$$

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